## The School of Science and Engineering

#### **Mathematics**

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#### **Professors**

John Dauns, Ph.D., Harvard

Lisa J. Fauci, Ph.D., New York

Pierre A. Grillet, Ph.D., Université de Paris

Morris Kalka, Ph.D., New York (Chair)

Slawomir Kwasik, Ph.D., University of Gdansk

Terry C. Lawson, Ph.D., Stanford

Michael W. Mislove, Ph.D., Tennessee

Victor H. Moll, Ph.D., New York

James T. Rogers, Jr., Ph.D., California, Riverside

Steven I. Rosencrans, Ph.D., M.I.T.

Frank J. Tipler, Ph.D., Maryland

Xuefeng Wang, Ph.D., Minnesota

Alexander D. Wentzell, Ph.D., Steklov Mathematical Institute

#### **Associate Professors**

Ricardo Cortez, Ph.D., California, Berkeley

Maurice J. Dupré, Ph.D., Pennsylvania

Alexander Kurganov, Ph.D., Tel-Aviv University

John Liukkonen, Ph.D., Columbia

Norbert Riedel, Ph.D., Technical University of Munich

Albert L. Vitter III, Ph.D., Princeton

David D. Yang, Ph.D., S.U.N.Y., Stony Brook

#### **Assistant Professors**

*Tai Huy Ha*, Ph.D., Queen's University, Ontario *Michelle Lacey*, Ph.D., Yale

#### **Emeritus**

Frank T. Birtel, Ph.D., Notre Dame (University Professor)

John E. Diem, Ph.D., Purdue

Laszlo Fuchs, Ph.D., University of Budapest

## MATHEMATICS REQUIREMENT FOR THE BACHELOR OF SCIENCE

Six credits of mathematics are required of all candidates for the bachelor of science. Any two mathematics courses (subject to the restrictions listed in Introductory Courses) except 115 may be used to satisfy this requirement. The combination of 115 and 116 may count as one course toward the B.S. degree requirement. Note that some departments recommend or require particular mathematics courses to supplement their majors. Students are therefore advised to consult their major department's listing in this catalog.

#### INTRODUCTORY COURSES

The following information is meant to aid students in planning their programs. It is not a substitute for the advice of the faculty advisors who are familiar with the student's individual situation.

- 111, 112 Not suitable for students who plan three semesters of calculus. Does not count toward the mathematics major.
- 115, 116 This course is intended for students without a strong background in mathematics. The material presented in Calculus 121 is covered in two semesters. This course will prepare the student to continue with the regular calculus sequence. If a student receives credit for 115 or 116, the student may not receive credit for 121. After finishing 115-116, the student can register for 122. A student may not register for this course after receiving credit for 121.
- 121, 122 Intended primarily for students with solid backgrounds in high school algebra, trigonometry, and precalculus. These courses are prerequisites for most upper division mathematics

courses. If a student has already received credit for 121, the student may not receive credit for 115 or 116. A student may not receive credit for both 122 and 131. 121 is a prerequisite for 122.

**131** Intended for students with prior knowledge of calculus. A score of 3 or higher in the Calculus AP test or permission of the undergraduate coordinator is required for admission. A student may not receive credit for both 122 and 131.

#### **AP Examination Credit Granted by the Department**

Subject	Score	Credit/Course(s)	
		Recommended Placement	
Calculus AB	5	4 credit hours, (121)	131
Calculus AB	4	4 credit hours, (121)	131
Calculus AB	3	no credit	131
Calculus BC	5	8 credit hours, (121, 122)	221
Calculus BC	4	8 credit hours, (121, 122)	221
Calculus BC	3	4 credit hours (121)	131

#### THE MAJOR PROGRAM

A major in mathematics consists of:

- (1) the following five core courses which are required of all mathematics majors: 121, 122, 221, 305, and 309; 115 and 116 may be substituted for 121; 131 may be substituted for 122.
- (2) four additional mathematics courses at the 300 level or above with the following provisos:
  - (a) one but not both of 217 and 224 may be substituted for one of the 300-level courses;
  - (b) at least one course must be at the 400 level or above;
  - (c) an advanced course in another department, with a high mathematical content, may, with the approval of the departmental undergraduate studies committee, be substituted for one of the 300-level courses.

(3) the year-long Senior Seminar, Mathematics 398, 399 is required of all mathematics majors who are not doing an Honors Project within the department. Students planning to graduate in December should begin this course in the fall of the preceding year. The Senior Seminar does not count towards the additional 300 level math courses in (2).

A student may receive either a B.A. or B.S. degree with mathematics as a major subject. A freshman should take the appropriate calculus course. Students with no prior calculus course should normally take 121 and 122 during the freshman year. Students with one semester of calculus credit (or equivalent knowledge) should take 131. Students with two semesters of calculus credit should start in 221 and contact a mathematics major advisor during the first semester for major program planning advice. It is also recommended that a prospective mathematics major take Computer Science 101 and Physics 131 and 132 during either the freshman or sophomore year.

Students should take the core courses as early as possible in their programs. After completing 221, the most frequent courses taken next are usually selected from the core courses 309, 305 and 217, 224, 301. It is generally recommended to take 309 before 305, but they can be taken concurrently. 309 is offered every semester and 305 is currently only offered in the fall semester, although it may be offered in the spring semester as well if there is sufficient demand. Each introduces the student to more theoretical mathematics than has been encountered in the calculus courses, and these courses provide the foundation for many advanced courses. The course 217 is required for computer science minors and majors. The course 224 gives an introduction to applied mathematics, and can be counted toward the major (although both 217 and 224 cannot both count). However, majors are advised to forego 224 and instead take 424 after taking 309. There is considerable overlap in 224 and 424, and both may not be taken for credit. The course 301 provides an introduction to probability and statistics, and it is a prerequisite to other courses in the area (602, 603, 604). It should be taken in the sophomore year by students interested in pursuing a concentration in statistics, which includes these four courses in addition to the core courses. Students considering a math major should arrange an appointment with the Director of the Major Program early in their program to get advice on course selection within the major.

The major program is designed to provide the student with a solid foundation during the first two years and provide for a variety of programs of study during the junior and senior years. A major program in mathematics can provide a background for both graduate study and work in a variety of areas of the mathematical sciences such as mathematics, applied mathematics, computer science, and statistics as well as provide preparation for professional schools such as law, medicine, and business. The major program should be designed as early as possible with the student's goal in mind and with the help of the major advisor.

#### MINOR IN MATHEMATICS

A minor in mathematics consists of 121, 122 or 131, 221, 309 and two additional courses at the 200 level or above, with the stipulation that 217 and 224 cannot both count toward the minor.

#### JUNIOR YEAR ABROAD

Students planning a major in mathematics and to participate in the Junior Year Abroad (JYA) program are advised to consult a major advisor when planning their sophomore programs. Participants in the JYA Program majoring in mathematics must successfully complete two courses in mathematics at the 300 level or above after returning from abroad.

#### **SPECIAL HONORS**

The purpose of the Honors Program in mathematics is to provide exceptional students with an opportunity to complete an intensive program in their major area and to receive recognition for that work. This program is recommended for students who wish to do graduate work in mathematics or related sciences. Honors in mathematics requires a 3.5 grade-point average in all courses taken in mathematics and in related courses serving to fulfill major requirements.

Honors also requires either (1) an honors project or (2) the completion of two additional upper-level mathematics courses with grades of B or higher. These can be selected from any course numbered 650 or higher or an honors version of a course numbered 300 or higher which has been approved in advance by the Undergraduate Studies Committee. An honors project must be under the supervision of a member of the mathematics faculty and receive prior approval from the student's major

advisor. The student receives six credits for the project, which is carried out over a two semester period. It must include a prospectus, a paper, and an oral presentation. Students wishing to complete an honors program must have their curriculum approved by their major advisor and the Undergraduate Coordinator during their junior year, and must declare their intention to complete an honors program at that time.

#### **MATHEMATICS PLACEMENT**

Information on placement may be obtained from the Department of Mathematics. Inquiries should be addressed to the Undergraduate Coordinator.

#### **HONORS SECTIONS**

Honors sections in 121, 122, 131, 221, and 224 are offered each year. Generally speaking, these sections treat the same material as the regular sections, but in greater depth. Honors sections of 300-level courses are offered when there is sufficient demand they are conducted in conjunction with a regular section but require extra work for honors credit. They may also include an additional class meeting each week. Admission to the honors sections is by invitation, but all interested students are encouraged to direct inquiries to the Undergraduate Coordinator.

#### **COURSES**

## MATH 111, 112 Probability and Statistics I, II (3, 3)

Staff. Prerequisite: high school algebra. Elementary probability theory with applications; random variables; distributions including a thorough discussion of the binomial, Poisson, and normal distributions; central limit theorem; histograms; sampling distributions; confidence intervals; tests of hypotheses; linear models; regression and correlation; chisquare test; non-parametric statistics; bioassay; sensitivity experiments; sequential test. 111 is a prerequisite for 112.

## Math 114 Statistics for Business (4)

Staff. Prerequisite: high school algebra. An introductory statistics course for pre-BSM students, which emphasizes basic topics on statistical inferences with extensive coverage of data collection and analysis as needed to evaluate the reported statistical results and to make good decisions in business. The course stresses the development of statistical thinking, the assessment of credibility and the value of the inferences made

from data, both by those who analyze and those who produce them. In comparison with Math 111, this course spends more time on statistical inference problems, less on probability, and teaches the use of Excel, a computer skill necessary nowadays in the business world.

## MATH 115, 116 Introduction to Calculus (3, 3)

Staff. The material of Calculus 121 is covered in two semesters, with diversions for topics in algebra, trigonometry, complex numbers as the need for these topics arises. Mathematics 115 is a prerequisite for 116. Students finishing the course sequence 115-116 may continue with 122 or any other course having Calculus 121 as a prerequisite. The combination of 115 and 116 may count as one course toward the B.S. degree requirement.

#### MATH 121 Calculus I (4)

Staff. Prerequisites: high school algebra, geometry, and trigonometry. Functions and their graphs, limits and continuity, derivatives and applications of derivatives, and introduction to the integral.

#### MATH 122 Calculus II (4)

Staff. Prerequisite: grade of at least C- in MATH 116 or 121. Integration; exponential, logarithmic, and trigonometric functions; techniques of integration; mean value theorem; Taylor's Theorem and Taylor series; and infinite series.

## MATH 131 Consolidated Calculus (4)

Staff. Prerequisite: a score of 3 or higher on the AB or BC Calculus AP test or permission of the mathematics department undergraduate coordinator. A combined course in Calculus I and II for students with a background in Calculus I. Students receive credit for both this course and 121 if they receive a Bor higher. Students may not receive credit for both 131 and 122.

## **MATH 217 Discrete Mathematics (3)**

Staff. Prerequisite: MATH 122 or 131. An introduction to the concepts and techniques of discrete mathematics including set theory, mathematical induction, graphs, trees, ordered sets, Boolean algebras, and the basic laws of combinatorics.

## MATH 221 Calculus III (4)

Staff. Prerequisite: MATH 122 or 131. A basic course in differential and integral calculus of several variables. Vectors in the plane and space. Vector functions, derivatives, arc length, curvature. Functions of several variables: continuity, partial derivatives, chain rule, gradient, optimization, Lagrange

multipliers. Double and triple integrals: change of variables, polar coordinates, cylindrical and spherical coordinates, surface area. Vector fields: gradient, curl, divergence, line and surface integrals, Green's, Stokes', and Divergence theorems.

#### **MATH 224 Introduction to Applied Mathematics (4)**

Staff. Prerequisite: MATH 122 or 131. An introduction to the techniques of applied mathematics. The emphasis will be on the mathematical modeling by differential equations of a variety of applications in the natural sciences. Numerical and graphical techniques for finding both quantitative and qualitative information about solutions will be discussed and implemented on the computer. No programming experience is assumed. Students may not receive credit for both 224 and 424.

#### MATH 300 Computational Problem Solving (4)

Staff. Prerequisite: MATH 121 or approval of instructor. This course will utilize a Scheme programming environment to teach student how to solve a broad range of mathematical problems.

#### MATH 301 Probability and Statistics (3)

Staff. Prerequisite: MATH 221. An introduction to statistics and the necessary probability background. Binomial, Poisson, and normal distribution; independence; sampling distribution; confidence intervals and hypothesis tests on the mean; variance; proportions; goodness of fit; contingency tables; linear regression.

## MATH 305 Real Analysis I (3)

Staff. Prerequisite: MATH 221. Introduction to analysis. Real numbers, limits, continuity, uniform continuity, sequences and series, compactness, convergence, Riemann integration. An indepth treatment of the concepts underlying calculus.

## MATH 309 Linear Algebra (4)

Staff. Prerequisite: MATH 221. An introduction to linear algebra emphasizing matrices and their applications. Gaussian elimination, determinants, vector spaces and linear transformations, orthogonality and projections, eigenvector problems, diagonalizability, Spectral Theorem, quadratic forms, applications. MATLAB is used as a computational tool.

## MATH 311 Abstract Algebra I (3)

Staff. Prerequisite: MATH 221. An introduction to abstract algebra. Elementary number theory and congruences. Basic group theory: groups, subgroups, normality, quotient groups, permutation groups. Ring theory: polynomial rings, unique

factorization domains, elementary ideal theory. Introduction to field theory.

## **MATH 313 Special Topics in Mathematics (3)**

Staff. Courses offered by visiting professors or permanent faculty. For description, consult department.

## **MATH 320 Combinatorics (3)**

Staff. Prerequisites: MATH 121, 122, and either 221 or 309 or approval of instructor. Basics of combinatorics with emphasis on problem solving. Provability, pigeonhole principle, mathematical induction. Counting techniques, generating functions, recurrence relations, Polya's counting formula, a theorem of Ramsey.

### MATH 331 Scientific Computing I (3)

Staff. Prerequisites: MATH 221, 224, or 424, and Computer Science 101 or equivalent. Errors. Curve fitting and function approximation, least squares approximation, orthogonal polynomials, trigonometric polynomial approximation. Direct methods for linear equations. Iterative methods for nonlinear equations and systems of nonlinear equations. Interpolation by polynomials and piecewise polynomials. Numerical integration. Single-step and multi-step methods for initial-value problems for ordinary differential equations, variable step size. Current algorithms and software.

## MATH 365 Number Theory and Applications (3)

Staff. The subject of number theory is one of the oldest in mathematics. The course will cover some basic material and describe interesting applications. One of the recurrent themes is the realization that mathematics that was developed usually for its own sake, has found applications in many unexpected problems. Some of the topics covered in the class are Pythagorean triples, prime numbers, divisibility and the highest common divisor, linear diophantine equations, congruences, round-robin tournaments and perpetual calendars, multiple functions, perfect numbers, primitive roots, pseudo-random numbers, decimal fractions and continued fractions, quadratic reciprocity.

## MATH 398-399 Seminar in Mathematics (1, 3)

Staff. Prerequisites: MATH 305, 309, and two additional courses at the 300-level or above. Under faculty guidance, students will select a topic in current mathematical research, write an expository article on that topic, and give an oral

#### MATH 406 Real Analysis II (3)

Staff. Prerequisites: MATH 305 and 309. An in-depth treatment of multivariable calculus. Extends the material covered in Mathematics 221. Chain rule, inverse and implicit function theorems, Riemann integration in Euclidean n-space, Gauss-Green-Stokes theorems, applications.

#### MATH 412 Abstract Algebra II (3)

Staff. Prerequisites: MATH 309 and 311. Abstract vector spaces, quotient spaces, linear transformations, dual spaces, determinants. Solvable groups. Field extensions, Galois theory, solvability of equations by radicals.

#### MATH 421 Differential Geometry (3)

Staff. Prerequisites: MATH 305 and 309. Theory of plane and space curves including arc length, curvature, torsion, Frenet equations, surfaces in three-dimensional space. First and second fundamental forms, Gaussian and mean curvature, differentiable mappings of surfaces, curves on a surface, special surfaces.

## **MATH 424 Ordinary Differential Equations (3)**

Staff. Prerequisite: MATH 309. Review of linear algebra, first-order equations (models, existence, uniqueness, Euler method, phase line, stability of equilibria), higher-order linear equations, Laplace transforms and applications, power series of solutions, linear first-order, systems (autonomous systems, phase plane), application of matrix normal forms, linearization and stability of nonlinear systems, bifurcation, Hopf bifurcation, limit cycles, Poincare-Bendixson theorem, partial differential equations (symmetric boundary-value problems on an interval, eigenvalue problems, eigenfunction expansion, initial-value problems in 1D). Students may not receive credit for both 224 and 424.

## MATH 430 Complex Analysis (3)

Staff. Prerequisite: MATH 305. The complex number system, complex integration and differentiation, conformal mapping, Cauchy's theorem, calculus of residues.

## MATH 441 Topology (3)

Staff. Prerequisite: MATH 305. An introduction to topology. Elementary point set topology: topological spaces, compactness, connectedness, continuity, homeomorphisms,

product and quotient spaces. Classification of surfaces and other geometric applications.

MATH 447 Analytical Methods of Applied Mathematics (3)

Staff. Prerequisites: MATH 221 and 224 or 424. Derivations of transport, heat/reaction-diffusion, wave, Poisson's equations; well-posedness; characteristics methods for first order PDE's; D'Alembert formula and conservation of energy for wave equations; propagation of waves; Fourier transforms; heat kernel, smoothing effect; maximum principles; Fourier series and Sturm-Liouville eigen-expansions; method of separation of variables, frequencies of wave equations, stable and unstable modes, long time behavior of heat equations; delta-function, fundamental solution of Laplace equation, Newton potential; Green's function and Poisson formula; Dirichlet Principle.

#### MATH 478 Introduction to Concurrency (3)

Staff. Prerequisites: MATH 217 and MATH 310 or approval of instructor. This course is a general introduction to Concurrency, i.e., the mathematical modeling of systems made up of several processes interacting with each other. The process algebra CSP (Communicating Sequential Processes) will be studied, both on the syntactic and semantic level. The denotational, operational, and algebraic models used to reason about the language will be presented, and examples will be used throughout to illustrate the theory.

## MATH 490 Advanced Topics in Mathematics (3)

Staff. Prerequisite: approval of instructor. Each section will have the specific topic listed as a subtitle and will have specific prerequisites at the 300 level or above. This course covers a variety of advanced topics in mathematics and exposes students to recent developments not available in other parts of the mathematics curriculum. It meets in conjunction with graduate level courses MATH 771-779. Topics covered will vary from semester to semester. Recent topics offered include Knot Theory and 3-Manifolds, Algebraic Combinatorics, Cardiac Modeling, Number Theory. Students may receive credit for MATH 490 more than once, when the topics covered are distinct.

#### **MATH 491, 492 Independent Studies (1-3, 1-3)**

Staff. Prerequisite: approval of the department. No more than four hours of 491-492 may be counted toward satisfying the major requirements.

#### **MATH H499-H500 Honors Thesis (3, 4)**

Staff. Prerequisite: approval of the department. Thesis may serve to satisfy part of the departmental honors requirements.

## MATH 602 Mathematical Statistics (3)

Staff. Prerequisites: MATH 301 and 221. Thorough review of key distributions for probability and statistics, including the multivariate calculus needed to develop them. Full derivation of sampling distribution. Classical principles of inference including best tests and estimations. Methods of finding tests and estimators. Introduction to Bayesian estimators.

#### MATH 603 Stochastic Processes (3)

Staff. Prerequisite: MATH 301. Markov processes, Poisson processes, queueing models, introduction to Brownian Notion.

#### MATH 604 Linear Models (3)

Staff. Prerequisites: MATH 301 and 309 or equivalent. Overview of multivariate analysis, theory of least squares linear regression, regression diagnostics, introduction to generalized linear models with emphasis on logistic regression. The student will complete several extended data analysis assignments using SAS, S-Plus, or R.

## MATH 624 Ordinary Differential Equations (3)

See MATH 424 for description.

#### MATH 635 Optimization (3)

Staff. Prerequisite: MATH 309 or equivalent. Constrained and unconstrained non-linear optimization; Linear programming, combinatorial optimization as time allows. Emphasis is on realistic problems whose solution requires computers, using Maple or Mathematica.

## MATH 651, 652 Topology I and II (3, 3)

Staff. Prerequisites: MATH 305 and 406. Point set topology. Connectedness, product and quotient spaces, separation properties, metric spaces. Classification of compact connected surfaces. Homotopy. Fundamental group and covering spaces. Singular and simplicial homology. Eilenberg-Steenrod axioms. Computational techniques, including long exact sequences. Mayer-Vietoris sequences, excision, and cellular chain complexes. Introduction to singular cohomology.

## MATH 655, 656 Differential Geometry I, II (3, 3)

Staff. Differential manifolds. Vector fields and flows. Tangent bundles. Frobenius theorem. Tensor fields. Differential forms,

Lie derivatives. Integration and deRham's theorem. Riemannian metrics, connections, curvature, parallel translation, geodesics, and submanifolds, including surfaces. First and second variation formulas, Jacobi fields, Lie groups. The Maurer-Cartan equation. Isometries, principal bundles, symmetric spaces, Kähler geometry.

## MATH 661, 662 Algebra I and II (3, 3)

Staff. Prerequisites: MATH 309 and 311. Vector spaces: matrices, eigenvalues, Jordan canonical form. Elementary number theory: primes, congruences, function, linear Diophantine equations, Pythagorean triples. Group theory: cosets, normal subgroups, homomorphisms, permutation groups, theorems of Lagrange, Cayley, Jordan-Hölder, Sylow. Finite abelian groups, free groups, presentations. Ring theory: prime and maximal ideals, fields of quotients, matrix and Noetherian rings. Fields: algebraic and transcendental extensions, survey of Galois theory. Modules and algebras: exact sequences, projective and injective and free modules, hom and tensor products, group algebras, finite dimensional algebras. Categories: axioms, subobjects, kernels, limits and colimits, functors and adjoint functors.

## MATH 665, 666 Differential Equations I, II (3, 3)

Staff. ODE: existence and uniqueness, stability and linearized stability, phase plane analysis, bifurcation and chaos. PDE: heat, wave, and Laplace equations, functional analytic (Sobolev space) and geometric (characteristic) methods. Maximum principle. Introduction to nonlinear PDE's.

## MATH 671, 672 Analysis I and II (3, 3)

Staff. Prerequisites: MATH 305, 309, and 406. Lebesgue measure on R. Measurable functions (including Lusin's and Egoroff's theorems). The Lebesgue integral. Monotone and dominated convergence theorems. Radon-Nikodym Theorem. Differentiation: bounded variation, absolute continuity, and the fundamental theorem of calculus. Measure spaces and the general Lebesgue integral (including summation and topics in R such as the Lebesgue differentiation theorem). L spaces and Banach spaces. Hahn-Banach, open mapping, and uniform boundedness theorems. Hilbert space. Representation of linear functionals. Completeness and compactness. Compact operators, integral equations, applications to differential equations, self-adjoint operators, unbounded operators.

## **MATH 675, 676 Computation I, II (3, 3)**

Staff. Floating point arithmetic (limitations and pitfalls). Numerical linear algebra, solving linear systems by direct and iterative methods, eigenvalue problems, singular value decompositions, numerical integration, interpolation. Iterative solution of nonlinear equations. Unconstrained optimization. Solution of ODE, both initial and boundary value problems. Numerical PDE. Introduction to fluid dynamics and other areas of application.

## MATH 681, 682 Applied Mathematics I and II (3, 3)

Staff. Prerequisites: MATH 305, 309, 347, and 406. Formulating mathematical models. Introduction to differential equations and integral equations. Fourier series and transforms, Laplace transforms. Generating functions. Dimensional analysis and scaling. Regular and singular perturbations. Asymptotic expansions. Boundary layers. The calculus of variations and optimization theory. Similarity solutions. Difference equations. Stability and bifurcation. Introduction to probability and statistics, and applications.

Note: Mathematics 651, 652, 655, 656, 661, 662, 665, 666, 671, 672, 675, 676, 681, 682 are the same as courses of the same title offered in the Graduate School. These courses are particularly recommended for students planning to do graduate work in mathematics.

# MATH 684 Numerical Methods in Partial Differential Equations (3)

Staff. Prerequisites: MATH 331 and 347 or approval of instructor. This course will present a detailed analysis of the methods for numerically approximating the solution of ordinary and partial differential equations typically encountered in applications from engineering and physics. Mathematical theory, practical implementation and applications will be emphasized equally. Typical applications to be discussed include population dynamics, particle dynamics, waves, diffusion processes.