Full- and Part-Time Wage Differentials and Female Labor Supply: Discontinuous Budget Constraint and Endogenous Wages

Myeong-Su Yun
Department of Economics
Tulane University
206 Tilton Hall
New Orleans, LA 70118
msyun@tulane.edu
phone: +1 504 862-8356
fax: +1 504 865-5869

January 31, 2003

Abstract

We introduce a structural model which jointly estimates the full-time wage premium and female labor supply, using the piecewise-linear budget constraint method. Our model incorporates a discontinuous budget line at cut-off hours (35 hours a week), caused by the coexistence of both full- and part-time wage rates, and makes wages fully endogenous to the labor supply choice. We estimate a structural model using the female sample from March 1995 current population survey. The estimate of the full-time premium from our structural model is slightly larger than that of OLS, while Heckman's two-step method predicts a small full-time wage premium. Our estimates for labor supply elasticities lie within the ranges reported in previous research.

JEL codes: J22, J31, C34

Keywords: full-time wage premium, endogenous wages, piecewise-linear budget constraint method, discontinuous budget line, female labor supply

The author wishes to thank Mark Killingsworth, Ira Gang, Hiroki Tsurumi, Roger Klein, Charles R. Romeo, Frank Vella, John P. Haisken DeNew, and participants of seminars at Rutgers University, SUNY-Albany, University of Colorado at Denver, Bates College, University of Western Ontario, McMaster University, Wilfrid Laurier University, Eastern Economic Association and Midwest Economics Association. He also gratefully acknowledges the financial support of the Peter Asch Award.

1 Introduction

This paper develops a structural model, based on the piecewise-linear budget constraint method, for jointly estimating the full- and part-time wage differentials and female labor supply.[1] In contrast to a standard labor supply model which assumes a single wage rate is offered to workers, our model assumes two distinct wage rates, full- and part-time wage rates, are offered.[2] The coexistence of two wage rates causes a significant change in the budget constraint workers face: the budget constraint becomes discontinuous. The discontinuous budget line makes the labor market participation and labor supply decisions very complicated. In this paper, we study wage differentials and the choice of labor force status by explicitly modeling the discontinuous budget line.

Early studies on full- and part-time wage differentials estimated the differentials using ordinary least square (OLS).[3] In recent years, most studies estimate the wage differentials using Heckman's two-step method in order to correct for the self-selection into full- or part-time jobs.[4] Studies using Heckman's two-step method first estimate the determinants of the choice between full- and part-time jobs using a reduced form labor supply function as an index (selection) function. Papers using the reduced form approach, however, may not be able to fully address the selection process when full- and part-time wage rates are offered.

We study the selection into different labor force statuses by explicitly modeling the budget constraint when full- and part-time wages are offered. Workers face a part-time wage rate if they work less than the cut-off hours of work ($\overline{H}$), and a full-time wage rate if they work more

[1]The piecewise-linear budget constraint method has been used to analyze the effects of taxes and transfers on labor supply. See Hausman (1980, 1985), and Moffitt (1986, 1990).

[2]For discussions on why full- and part-time wage rates are offered, see Blank (1990) and Averett and Hotchkiss (1997). See Killingsworth (1983) for the standard labor supply model.

[3]Owen (1978), Long and Jones (1981), and Ehrenberg, Rosenberg, and Li (1988).

[4]Simpson(1986), Leeds (1990), Hotchkiss (1991), Harris (1993), and Ermisch and Wright (1993). Blank (1990) has two indices, choice for participation and choice for full- or part-time job, and uses a maximum likelihood estimation method.

than or equal to the cut-off hours.[5] The budget line is discontinuous at the cut-off hours (see Figure 1), because full-time workers receive the full-time wage rate for all hours they work, rather than receiving the part-time wage rate for the first $\overline{H}$ hours and the full-time wage rate for additional hours (Averett and Hotchkiss (1997, p. 461)).

Averett and Hotchkiss (1997) study the choice of labor force statuses and labor supply when the budget line is discontinuous because full- and part-time wages are offered. However, they use the predicted full- and part-time wage rates and treat these wage rates as nonstochastic when they study the selection of labor force statuses.[6]

We develop a structural model which estimates both labor supply (choice of labor force status) and wages (full-time wage premium) jointly while making the wages fully endogenous. Our model extends the standard leisure-consumption choice model by introducing a discontinuous budget line caused by the two "endogenous" wage rates. We, however, ignore other factors complicating the labor supply decision.[7]

In section 2, the structural model is developed. We explain the choice of labor force status when the budget line is discontinuous due to the coexistence of the full- and part-time wage rates. Section 3 provides the econometric framework for estimating the full-time wage premium and labor supply jointly using an explicit utility function. Section 4 describes the data used in this study and discusses the empirical results of estimation of wages and labor supply. In section 5, we summarize the findings and contributions of this paper to the study of wages and labor supply.

---

[5]The cut-off point in our study is 35 hours of work per week following the official definition of the part-time work by Bureau of Labor Statistics.

[6]Heckman and MaCurdy (1981) highlight the importance of accounting for the presence of unobserved components of wages for the estimation of labor supply. See also Barzel (1973), Moffitt (1984), Lundberg (1985), and Tummers and Woittiez (1991) for labor supply models with endogenous wages.

[7]We do not take account of the effects of taxes, transfers, or time and money costs associated with working; they are left for future work.

2 Choice of Labor Force Status with a Discontinuous Budget Line

In this section, we discuss an individual's choice of labor force status including the participation decision.[8] When full- and part-time wage rates are offered, an individual may choose to work full- or part-time, or to work at the kink point (working $\overline{H}$ hours), or not to work. The choice of labor force status is determined according to individual's utility maximization.

Let a direct utility function, $U(H, C)$, be well defined over leisure (L) or labor (H), and consumption (C). Both leisure and consumption are assumed to be normal goods. It is convenient to assume that workers face full- and part-time wage rates over the whole range of labor supply. This assumption (along with the absence of taxes, transfers, and time and money costs associated with working) guarantees that conventional utility maximization leads to a tangency equilibrium at each wage rate. We can then easily derive an indirect utility function and an optimal labor supply function at each wage rate. The indirect utility function at full- and part-time wage rates are denoted as $V(W_f, N)$ and $V(W_p, N)$, respectively, where W_f, W_p, and N are full- and part-time wage rates, and non-labor income, respectively. The optimal labor supply at full- and part-time wage rates are H_f^* and H_p^*, respectively.

In addition, in order to compare utilities, we need following two indices,

$$\mathbf{I}_1 = V(W_p, N) - U(\overline{H}, C_f^{\overline{H}}), \tag{1}$$

$$\mathbf{I}_2 = U(0, N) - U(\overline{H}, C_f^{\overline{H}}), \tag{2}$$

where $C_f^{\overline{H}} = W_f \cdot \overline{H} + N$. Index 1, shown in Figure 1 (A), compares utility attained by

[8]Participation is usually defined to include employment and unemployment. However, most studies of hours of work do not count the unemployed in the definition of participation. The definition of the market participant frequently used in studies of hours of work is that the person worked for money some time in the survey year (month or week). Blundell, Ham and Meghir (1987) is a rare exception. They include unemployment in the definition of participation. We treat unemployment as non-participation to keep the analysis simple.

working H_p^* at W_p with utility attained by working $\overline{H}$ at W_f. Index 2, shown in Figure 1 (B), compares utility at the kink point vs. the utility of not working.

The choice of labor force status when full- and part-time wage rates are offered is summarized in Table 1. An individual does not participate into labor market either because her reservation wage rate is higher than a full-time wage rate ($W_r > W_f$) or because the utility of not working is greater than the utility of working at the kink point, when her reservation wage rate is between the full- and part-time wage rates ($W_p < W_r \leq W_f$ and $\mathbf{I}_2 > 0$).

Once an individual is participating in the labor market ($W_r \leq W_p$, or $W_p < W_r \leq W_f$ and $\mathbf{I}_2 \leq 0$), then the choice of labor force status is determined by optimal labor supply (H_f^* and H_p^*) and index 1. An individual chooses a full-time job if H_f^* is greater than the cut-off hours ($H_f^* > \overline{H}$). An individual chooses a part-time job if both H_f^* and H_p^* are less than or equal to the cut-off hours and the utility of working part-time at tangency equilibrium is greater than the utility of working at the kink point ($0 < H_f^* \leq \overline{H}$, $0 < H_p^* \leq \overline{H}$, and $\mathbf{I}_1 > 0$). In other cases, the person chooses working at kink point.[9]

In this section, we have shown the choice of the labor force status when workers face the discontinuous budget constraint caused by full- and part-time wage rates. Our discussion in this section is independent of a functional form of the utility function. For empirical study, however, the selection rules presented in Table 1 will need to be written in terms of an explicit utility function and/or the labor supply function. In next section, we will specify utility function we use to find the choice of labor force status, and discuss a methodology for jointly estimating wages and labor supply functions.

[9]There are three cases: Case 1. $0 < H_f^* \leq \overline{H}$ and $H_p^* > \overline{H}$, Case 2. $0 < H_f^* \leq \overline{H}$, $0 < H_p^* \leq \overline{H}$, and $\mathbf{I}_1 \leq 0$, and Case 3. $0 < H_f^* \leq \overline{H}$, $H_p^* \leq 0$, $W_p < W_r \leq W_f$ and $\mathbf{I}_2 \leq 0$.

3 Econometric Specification for Jointly Estimating Wages and Labor Supply

3.1 Basic Specification

The following direct utility function leads to the linear labor supply function and has been widely used in labor supply studies:[10]

$$U(H,C) = \frac{1}{\alpha - \delta H} \exp\left[-\frac{\delta(H - X\beta - \delta C - \epsilon)}{\alpha - \delta H}\right], \tag{3}$$

where H, C, W, and N are hours of work, consumption, the wage rate, and non-labor income, respectively, and ϵ is a taste shifter variable that accounts for individual heterogeneity.

The budget constraint is

$$C = WH + N, \tag{4}$$

while the wage equation is

$$W = \begin{cases} Z\gamma + \nu & \equiv \widehat{W}_p + \nu, \quad \text{if } 0 < H < \overline{H}, \\ Z\gamma + \Pi + \nu & \equiv \widehat{W}_f + \nu, \quad \text{if } H \geq \overline{H}, \end{cases} \tag{5}$$

where Π is the full-time wage premium, assumed positive.[11] $\widehat{W}_i$ and ν are respectively the deterministic and stochastic parts of the wage equation, where $i = p, f$ for part- and full-time wage rates, respectively. It is possible to impute two wages because the unobserved component of wages (ν) is assumed to be unique to each person.

The optimal hours of work at the full- and part-time wage rates (H_f^* and H_p^*, respectively)

[10]For example, Moffitt (1983, 1984) and Burtless and Moffitt (1984, 1985).

[11]This specification for wages is used in Blank (1990) and Ehrenberg, Rosenberg, and Li (1988).

are

$$
(6)\qquad \begin{aligned} H_i^* &= \alpha W_i + X\beta + \delta N + \epsilon \\ &= \underbrace{\alpha \widehat{W_i} + X\beta + \delta N}_{\widehat{H_i}} + \underbrace{\alpha\nu + \epsilon}_{\eta}\,, \qquad i = f, p, \end{aligned}
$$

where $\widehat{H_i}$ and η ($\equiv \alpha\nu + \epsilon$) are the deterministic and stochastic components of the optimal hours of work, respectively. We impose Slutsky condition ($\alpha - \delta H_i^* > 0$), which rules out Kink 1 in Table 1 (Stern (1986)). In equation (6), wages are treated as endogenous, not exogenous.

The indirect utility function derived by solving worker's utility maximization problem is ,

$$
(7)\qquad V(W_i, N) = \frac{1}{\alpha - \delta H_i^*} \exp(-\delta W_i), \qquad i = f, p.
$$

Now we can define our two indices for comparing utilities ($\mathbf{I}_1$, $\mathbf{I}_2$) in natural logarithms form. After some algebraic manipulation, the indices become:

$$
(8)\qquad \begin{aligned} I_1 &= \log V(W_p, N) - \log U(\overline{H}, C_f^{\overline{H}}) \\ &\approx \underbrace{-\log(\frac{\alpha - \delta\widehat{H_p}}{\alpha - \delta\overline{H}}) + \frac{\delta(\overline{H} - \widehat{H_p} - \delta\overline{H}\Pi)}{\alpha - \delta\overline{H}}}_{\mathcal{A}} \\ &- \underbrace{\frac{\delta^2(\overline{H} - \widehat{H_p})}{(\alpha - \delta\widehat{H_p})(\alpha - \delta\overline{H})}}_{\mathcal{B}}(\alpha\nu + \epsilon) \\ &+ \underbrace{\frac{\delta^2}{2(\alpha - \delta\widehat{H_p})^2}}_{\mathcal{C}}(\alpha\nu + \epsilon)^2, \end{aligned}
$$

and

$$
(9) \qquad \begin{aligned} I_2 &= \log U(0, N) - \log U(\overline{H}, C_f^{\overline{H}}) \\ &= \underbrace{-\log(\frac{\alpha}{\alpha - \delta\overline{H}}) + \frac{\delta\overline{H}(\alpha - \delta\widehat{H_f})}{\alpha(\alpha - \delta\overline{H})}}_{\mathcal{D}} \\ &- \underbrace{\frac{\delta^2\overline{H}}{\alpha(\alpha - \delta\overline{H})}}_{\mathcal{E}} (\alpha\nu + \epsilon). \end{aligned}
$$

We use a second-order Taylor expansion with respect to ϵ and ν around zero to approximate index 1, equation (8), because closed-form solutions for ϵ and ν of index 1 do not exist.[12] Index 1 has two roots, $UR = (\mathcal{B} + \sqrt{\mathcal{B}^2 - 4\mathcal{AC}})/2\mathcal{C}$ and $LR = (\mathcal{B} - \sqrt{\mathcal{B}^2 - 4\mathcal{AC}})/2\mathcal{C}$, where $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are defined in equation (8).

The utility of working part-time, $V(W_p, N)$, is greater than that of working at the kink point, $U(\overline{H}, C_f^{\overline{H}})$, when η is greater than UR or smaller than LR, while $V(W_p, N)$ is smaller than $U(\overline{H}, C_f^{\overline{H}})$ when η lies between LR and UR. The upper root is, however, irrelevant to the choice of working part-time because workers always prefer working full-time to working part-time when η is greater than UR.

The sign of $\mathcal{C}$ is positive assuming $\delta \neq 0$ and $\alpha - \delta\widehat{H}_p > 0$ (a version of Slutsky condition). However the algebraic form does not help us determine the magnitude of LR and UR. Compared to index 1, it is easy to derive index 2, equation (9).[13] We may expect the sign of $\mathcal{E}$ is likely to be positive assuming $\alpha > 0$, $\delta \neq 0$, and $\alpha - \delta\overline{H} > 0$ (another version of Slutsky condition).[14]

In order to reduce the burden of computation, we adopt a Tobit type specification which removes explicit reference to the reservation wage rate from the model specification.[15] If

[12] Averett and Hotchkiss (1997) use a numerical method to find roots of equation (see also Hausman (1980)).

[13] Averett and Hotchkiss (1997) ignore this index.

[14] α is assumed positive because the sign of the ratio inside log in $\mathcal{D}$ of equation (9) should be positive. The denominator of the ratio is assumed positive by applying the Slutsky condition.

[15] By eliminating reservation wage rates from our specification, we actually impose a restriction that the

the reservation wage rate is higher than the full-time (part-time) wage rate, the optimal hours of work at the full-time (part-time) wage rate, H_f^* (H_p^*), are less than zero. When the reservation wage rate is between the full- and part-time wage rates, then H_f^* is less than zero while H_p^* is positive.

In short, we use two indices for utility comparison (I_1, I_2) and two optimal hours of work (H_f^*, H_p^*) to analyze the labor supply decision in the presence of a discontinuous budget line. The selection rules for choosing the optimal labor supply (H^*) may be summarized as follows:

$$H^* = \begin{cases} H_f^* & \text{if } \eta > \overline{H} - \widehat{H_f}, \\ H_p^* & \text{if } -\widehat{H_p} < \eta \leq \min\left(\overline{H} - \widehat{H_f},\, LR\right), \\ \overline{H}_{\text{Kink A}} & \text{if } \max\left(-\widehat{H_p},\, LR,\, \frac{\mathcal{D}}{\mathcal{E}}\right) < \eta \leq \overline{H} - \widehat{H_f}\ , \\ \overline{H}_{\text{Kink B}} & \text{if } \max\left(-\widehat{H_f},\, \frac{\mathcal{D}}{\mathcal{E}}\right) < \eta \leq \min\left(\overline{H} - \widehat{H_f},\, -\widehat{H_p}\right), \\ 0 & \text{if } \eta \leq \min\left(\frac{\mathcal{D}}{\mathcal{E}},\, -\widehat{H_p}\right)\ , \end{cases} \tag{10}$$

where Kink A and B are Kink 2 and Kink 3 in Table 1, respectively.

If the observed hours of work (H) are assumed to be the optimal labor supply (H^*), the selection rules predict that 1) bunching in hours of work should occur at the cut-off hours, and 2) part-time workers will not choose hours of work that are close to cut-off hours. However, the data do not support these two predictions.

3.2 Adding Measurement Error

A measurement error is introduced to fill the gap between the prediction from theory of the previous section and women's actual labor supply.[16] Labor supply with a measurement error

labor force participation and hours decisions are strongly tied.

[16]Adding a measurement error to labor supply is the standard solution in the literature on the piecewise-linear budget constraint method. See Moffitt (1986) and MaCurdy, Green and Paarsch (1990). The measurement error is interpreted as an "optimization" error that reflects the degree to which the observed hours of

is denoted as $H_m = H^* + e$, where H^* is the optimal hours of work, defined in equation (10), and e is a measurement error.

Observed labor supply (H) is positive only when optimal hours of work are positive ($H^* > 0$) and labor supply with measurement error is also positive ($H_m > 0$). Observed hours of work (H) are zero when either desired hours are zero ($H^* = 0$), or a low realization of a measurement error (e) causes labor supply with measurement error to be non-positive ($H_m \leq 0$) though the optimal labor supply is positive ($H^* > 0$).

Once measurement error is added, optimal hours of work (H^*) are not observed. Since we cannot observe the true choice of labor supply, the likelihood of labor supply should be equal to the sum of the probabilities over all possible true labor force statuses. We estimate hours and wages jointly, both for the whole sample and for only working women, by maximizing likelihood functions. The likelihood function for the whole sample is [17]

$$L = \prod_{H>0} Pr(H, W|H > 0) \cdot Pr(H > 0) \prod_{H=0} Pr(H = 0), \tag{11}$$

and the likelihood function for the working sample is

$$L = \prod_{H>0} Pr(H, W|H > 0), \tag{12}$$

where $Pr(H, W|H > 0) = Pr(H, W)/Pr(H > 0)$ since $Pr(H, W, H > 0) = Pr(H, W)$, and $Pr(H, W|H > 0) \cdot Pr(H > 0) = Pr(H, W)$; $Pr(H, W)$, $Pr(H > 0)$, and $Pr(H = 0)$ are, respectively,

$$Pr(H, W) = \sum_j Pr\left(H^* = j, H = j + e, W = \widehat{W}_i + \nu\right), \tag{13}$$

$$Pr(H > 0) = \sum_j Pr\left(H^* = j, H > 0\right), \tag{14}$$

work (H) deviate from worker's optimal hours of work (H^*) (MaCurdy, Green and Paarsch (1990, p. 434)).

[17] See appendix for the functional specifications of the likelihood functions.

and

$$Pr(H=0)=\sum_{j} Pr\left(H^{*}=j, H=0\right), \tag{15}$$

where $j = H_f^*$, H_p^*, $\overline{H}_{\mathrm{Kink\,A}}$, and $\overline{H}_{\mathrm{Kink\,B}}$ for $Pr(H, W)$ and $Pr(H > 0)$, and $j = H_f^*$, H_p^*, $\overline{H}_{\mathrm{Kink\,A}}$, $\overline{H}_{\mathrm{Kink\,B}}$, and 0 for $Pr(H = 0)$; $Pr(H^* = 0, H = 0) = Pr(H^* = 0)$; Since our model does not contain a reporting error, $\widehat{W_i}$ means $\widehat{W_f}$ ($\widehat{W_p}$) if the workers are "observed" as full-time (part-time) workers. This means that the value of ν is unique for individual regardless of her preferred labor force status.

4 Estimation of Wages and Labor Supply

In this section, we present our estimates for wages and hours from the structural model developed earlier, using the whole sample and the sample of workers. The estimation is implemented using the SAS non-linear programming (NLP) procedure (SAS Institute, 1997).

4.1 Data

We use a sample of women drawn from the March 1995 current population survey (CPS), mainly using the responses to questions about the survey month.[18] Hence the data comes from the outgoing rotation group only.[19] The sample includes females aged 25 to 60 who were not in school, retired, disabled or self-employed. We include married women only if their spouses are aged 25 or more. We exclude observations if the female receives either less than \$3 per hour or more than \$40 per hour, or if she works less than 5 hours or more than

[18]Information on last year's earnings is used to compute non-labor income.

[19]Avernett and Hotchkiss (1997) also use outgoing rotation groups only. However, Blank (1990) uses the whole working sample, since she uses information related to last year's labor market activity.

75 hours a week.[20] The shares of full- and part-time workers and non-working women in our sample are, respectively, 65%, 15%, and 20%.

Table 2 describes the variables used for our study and Table 3 shows means and standard deviations of variables used in the analysis. The characteristics of working women are different from those of non-working women. Working women are older and have more years of education than non-working women. The proportion of whites in labor force is greater than that in non-working. Non-working women have a higher marriage rate, have more children (both under age 6 and between age 6 and 18), and have larger family size. Non-working women have a higher non-labor income (including husband's earnings) than that of working women, which may be related to higher marriage rates because non-labor income excluding husband's earnings is not much different between non-working women and working females. Non-working women live in metropolitan areas more than working women.

Part-time workers are older than non-working people, but not much younger or older than full-time workers. Full-time workers have achieved more years of education and non-working people have achieved less education than part-time workers. Part-time workers have the highest proportion of whites while non-working people have the lowest proportion of whites. Part-time workers have the highest rate of marriage, while full-time workers have the lowest rate of marriage. Non-working people have more children under age 6 than other two groups while part-time workers have more children between age 6 to 18 than full-time and non-working women. Non-working women have the largest family size and full-time workers have the smallest family size. Full-time workers have the lowest non-labor income, while both part-time workers and non-working women have higher non-labor income. This may be related to the low marriage rate of full-time workers, since non-labor income without husband's wages are not significantly different from one another.

[20] After excluding observations who are in school, retired, disabled or self-employed, we have 4746 observations. We lose 12 observations due to our husband's age restriction (aged 25 or older), 13 observations due to the hours restriction, and 47 observations due to the wage restriction. The restrictions on wages and hours are imposed in order to facilitate estimation. As noted in Moffitt (1986, p. 324), the maximization of the likelihood function of our model is fairly difficult, especially using the whole sample.

4.2 Empirical Findings

Table 4 shows the estimates of wages and labor supply from our structural model.[21] The second and third columns of the Table 4 are the results using the whole and only the working samples from maximizing likelihood functions, equations (11) and (12), respectively. We call our structural model measured using the whole and only the working samples as DBL (discontinuous budget line) and conditional DBL model, respectively, following the terminology for the Tobit model by Mroz (1987). We can modify our DBL model to estimate the labor supply model with exogenous wages as Averett and Hotchkiss (1997) do. We use the estimates for wages from our DBL models to compute predicted wages. The results, not reported in the table, are close to those of our DBL models.

We will discuss the results focusing on two aspects: the full-time wage premium and elasticities of labor supply. For full-time wage premium, we will compare our estimates with those using Heckman's two-step method used by most previous papers.[22] For the elasticities of labor supply, we compare our estimates with those from Tobit models.[23]

Though the signs of the significant estimates of hours and wages from the DBL models are reasonable, the DBL model (but not the conditional DBL model) produces very large estimates for full-time wage premium and labor supply parameters, especially those for wages and non-labor income. This may arise from the Tobit type specification we adopted, which imposes strong ties between participation and hours equations. The larger estimates of the Tobit model relative to those of conditional Tobit have already been noted in previous studies (see Mroz (1987) and Zabel (1993)).[24] This may indicate that the DBL model using

[21]We do not include education in the hours equation when we estimate our model. The omission of education variable is not rare in simultaneous equations model. Hausman and Wise (1977, p. 931) assume that "these attributes (education, I.Q., and occupation training) of individuals, given their wage rates, do not affect their choices between labor and leisure." Blundell, Duncan and Meghir (1992) cite identification as a reason why education and other variables are excluded from the hours equation.

[22]For Heckman's two-step estimation method, see Heckman (1979). For the extension to double selection rules, see Fishe, Trost and Lurie (1981), Ham (1982), and Tunali (1986).

[23]See Heckman (1974), Wales and Woodland (1980), Mroz (1987), and Zabel (1993) for Tobit model.

[24]Mroz (1987, p. 790) concludes "the hours of work decisions made when the woman is in the labor force

the whole sample may not be a good model for jointly estimating the full-time wage premium and labor supply.

Table 5 summarizes full-time wage premium estimated using various models. The OLS estimate of the full-time wage premium is \$2.22. When the (level) wage rates are regressed on exogenous variables including the selection bias correction terms (λ's) using Heckman's two-step method, the estimates of the full-time wage premium are not significantly different from zero and much smaller than the OLS estimate. This finding is consistent with previous studies.[25]

Previous studies conclude, based on results similar to ours, that OLS overestimates the wage premium for full-time jobs. They further claim that behavioral and skill differences account for most of the wage differential between full- and part-time jobs. However, Heckman's two-step method, using reduced form indices, does not fully account for the choice of the labor force status when the budget line is discontinuous due to full- and part-time wages.

The values of the wage premium for the working full-time from our DBL model are \$4.58 (DBL), and \$2.83 (conditional DBL). The estimates for the full-time wage premium are significant from the t-test for both DBL models.[26] In both the DBL and the conditional DBL models, the estimates for full-time wage premium are quite different from those found in previous papers using Heckman's two-step method. One might conjecture that the estimation using Heckman's two-step method tends to shrink the full-time wage premium in order to support the part-time workers who work close to 35 hours (cut-off hours). This is because

appear quite distinct from her labor force participation decision."

[25]When the log-wage rates are used as a dependent variable, the OLS estimates for the full-time premium is 0.23. The estimates using Heckman's two-step method are 0.01 (probit selection), 0.11 (ordered probit selection), and 0.05 (bivairate selection), and not significantly different from zero except the estimate of ordered probit selection model.

[26]We can test the null hypothesis of no wage premium for working full-time using a likelihood ratio test. See Amemiya (1985, pp. 141-146) for the likelihood ratio test. For the test, a DBL model under the null hypothesis is estimated. The likelihood ratio test statistic for the zero premium hypothesis is 3318.82 ($= 2 * (28728.45 - 27069.04)$) when the whole sample is used. The null hypothesis ($\Pi = 0$) is rejected because the table value for the test, with one degree of freedom, is 3.842. However, we cannot do the likelihood ratio test when we use only the working sample. This is because the conditional DBL model is under-identified when the full-time wage premium is zero.

Heckman's two-step method assumes the observed hours of work are the desired hours by workers and overlooks the importance of the discontinuous budget line. Previous papers based on Heckman's two-step method may underestimate women's willingness to accept longer work hours or inflexible work schedules (full-time work) in return for higher wages (full-time wage premium) by equating the observed labor supply to female worker's true choice. Our DBL model assumes that the preferred labor supply of workers may differ from the observed labor force status. This assumption allows the full-time wage premium to move freely according to women's unobserved preferences.

Table 6 summarizes elasticities of labor supply implied by the coefficients of labor supply.[27] The uncompensated wage elasticity is measured using $\alpha \cdot W/H$ and the total income elasticity is measured using $\delta \cdot W$, where α and δ are the coefficients of labor supply parameter for wages and non-labor income, respectively. The elasticities have the expected signs, except for total income elasticity measured from the conditional DBL model, and their magnitudes lie within the ranges reported in previous papers.[28] As already noted above, the Tobit and the DBL models which use the whole sample imply huge elasticities. We can also observe that the supply of observed part-time workers ($0 < H < 35$) is more elastic than that of observed full-time workers ($H \geq 35$).

The elasticities from our DBL models, however, should be interpreted with caution. "These elasticities are no longer very meaningful when the budget constraint is nonlinear...," (Moffitt (1984, p. 561)) because the usual comparative statics of labor supply no longer hold when the budget set is nonconvex, as in our DBL model. For example, the increase in wages (non-labor income) may reduce (increase) labor supply; we can picture the situation where women who previously worked at the kink may work part-time due to the increase in wages, or the situation where women who previous worked part-time may work at the kink due

[27]Previous studies of the elasticity of female labor supply are summarized in Killingsworth and Heckman (1986) and Killingsworth (1983).

[28]The unexpected sign of total income elasticity measured from the conditional DBL model may be of little importance since the estimate for non-labor income in hours equation(δ) from the conditional DBL model is not significant.

to the increase in the non-labor income. Also, the small change in wages may change the hours of work a lot because the jump from working part-time to working at the kink, or vise versa, is possible. Considering this possibility, the statement by Averett and Hotchkiss (1997, pp. 467-468) saying that "by explicitly incorporating the part-time/full-time wage differential into the structure of the model, we are not forcing the elasticity to capture large wage changes that would occur as hours move from part-time to full-time" does not apply to our results. Their statement may describe a case among many possible situations where labor supply with a discontinuous budget line has smaller elasticities than those of Tobit and OLS, which is the opposite of our finding.[29] The size of elasticities seems to be a matter for empirical studies, and these cannot be determined *a priori*.

5 Conclusion

This paper extends conventional labor supply models by allowing two wage rates, hereby recognizing the widely accepted view that part-time workers may receive a lower wage rate than do equally qualified full-time workers. The coexistence of full- and part-time wage rates provides us with a challenge to estimate wage differentials and labor supply. By examining the implications of the coexistence of the two wage rates, we conclude that the budget line is discontinuous. We develop a structural model for jointly estimating full- and part-time wage differentials and labor supply by explicitly modeling the discontinuous budget line and making wages fully endogenous to the choice of labor force status.

Our estimates of the full-time wage premium do not confirm the literature's apparent consensus that OLS overestimates the full-time wage premium. Our results are quite different from those of previous studies based on Heckman's two-step method which have not modeled the budget constraint explicitly. Our findings on the full-time wage premium highlight the

[29]The estimates obtained by modifying our DBL model to estimate labor supply with exogenous wages are similar to those from our DBL model, which means that the elasticities are also similar to those from our DBL model.

importance of explicitly modeling the budget constraint when the budget constraint is not continuous. We also learn that labor supply becomes more elastic when two wages are offered compared to either a Tobit model with single wage rate or OLS. We may conclude that the magnitudes of the labor supply elasticities are matter for empirical studies, and these cannot be determined *a priori*.

This study opens the possibility of exploring the discontinuous budget line not only in the model of full- and part-time work but also in other areas. For the study of labor supply, it also provides one possible way to make wages fully endogenous to the choice of labor force status in a piecewise-linear budget constraint model.

REFERENCES

Amemiya, Takeshi (1985): *Advanced Econometrics*. Cambridge: Harvard University Press.

Averett, Susan L. and Julie L. Hotchkiss (1997): "Female Labor Supply with a Discontinuous, Nonconvex Budget Constraint: Incorporation of a Part-time/Full-time Wage Differential," *Review of Economics and Statistics*, 76, 461-470.

Barzel, Yoram (1973): "The Determination of Daily Hours and Wages," *Quarterly Journal of Economics*, 87, 220-238.

Blank, Rebecca M. (1990): "Are Part-Time Jobs Bad Jobs?," in *A Future of Lousy Jobs?: The Changing Structure of U.S. Wages*, ed. by Gary Burtless. Washington D.C.: Brookings Institution, 123-155.

Blundell, Richard, Alan Duncan and Costas Meghir (1992): "Taxation in Empirical Labour Supply Models: Lone Mothers in the UK," *Economic Journal*, 102, 265-278.

Blundell, Richard, John Ham, and Costas Meghir (1987): "Unemployment and Female Labour Supply," *Economic Journal*, 97, 44-64.

Burtless, Gary and Robert A. Moffitt (1984): "The Effect of Social Security Benefits on the Labor Supply of the Aged," in *Retirement and Economic Behavior*, ed. by Henry J. Aaron and Gary Burtless. Washington D.C., Brookings Institution, 135-171.

——— (1985): "The Joint Choice of Retirement Age and Postretirement Hours of Work," *Journal of Labor Economics*, 3, 209-236.

Ehrenberg, Roland G., Pamela Rosenberg and Jeanne Li (1988): "Part-Time Employment in the United States," in *Employment, Unemployment and Labor Utilization*, ed. by Robert A. Hart. Winchester, M.A.: Unwin Hyman, 256-281.

Ermisch John F. and Robert E. Wright (1993): "Wage Offers and Full-Time and Part-Time Employment by British Women," *Journal of Human Resources*, 28, 111-133

Fishe, Raymond P.H., Robert P. Trost, and Philip M. Lurie (1981): "Labor Force Earnings and College Choice of Young Women: An Examination of Selectivity Bias and Comparative Advantage," *Economics of Education Review*, 1, 169-191.

Ham, John C. (1982): "Estimation of Labour Supply Model with Censoring Due to Unemployment and Underemployment," *Review of Economic Studies*, 49, 335-354.

Harris, Richard D. (1993): "Part-Time Female Earnings: An Analysis using Northern Ireland NES Data," *Applied Economics*, 25, 1-12.

Hausman, Jerry (1980): "The Effect of Wages, Taxes, and Fixed Costs on Women's Labor Force Participation," *Journal of Public Economics*, 14, 161-194.

——— (1985) "The Econometrics of Nonlinear Budget Sets," *Econometrica*, 53, 1255-1282.

Hausman, Jerry A. and David A. Wise (1977): "Social Experimentation, Truncated Distributions, and Efficient Estimation," *Econometrica*, 45, 919-938.

Heckman, James (1974): "Shadow Prices, Market Wages, and Labor Supply," *Econometrica*, 42, 679-694.

——— (1979): "Sample Selection Bias as a Specification Error," *Econometrica*, 47, 153-161.

Heckman, James J. and Thomas E. MaCurdy (1981): "New Methods for Estimating Labor Supply Functions: A Survey," *Research in Labor Economics*, 4, 65-102.

Hotchkiss, Julie L. (1991): "The Definition of Part-Time Employment: A Switching Regression Model with Unknown Sample Selection," *International Economic Review*, 32, 899-917.

Killingsworth, Mark R. (1983): *Labor Supply*, Cambridge: Cambridge University Press.

Killingsworth, Mark R. and James J. Heckman (1986): "Female Labor Supply: A Survey," in *Handbook of Labor Economics, Vol. I*, ed. by Orley Ashenfelter and Richard Layard. Amsterdam: Elsevier Science B.V., 103-204.

Leeds, Michael A. (1990): "Part-Time Status and Hourly Earnings of Black and White Men," *Economic Inquiry*, 33, 544-554.

Long, James E., and Ethel B. Jones (1981): "Married Women in Part-Time Employment," *Industrial and Labor Relations Review*, 34, 413-425.

Lundberg, Shelly (1985): "Tied Wage-Hours Offers and the Endogeneity of Wages," *Review of Economics and Statistics*, 67, 405-410.

MaCurdy, Thomas, David Green, and Harry Paarsch (1990): "Assessing Empirical Approaches for Analyzing Taxes and Labor Supply," *Journal of Human Resources*, 25, 415-490.

Moffitt, Robert (1983): "An Economic Model of Welfare Stigma," *American Economic Review*, 73, 1023-1035.

——— (1984): "The estimation of a Joint Wage-Hours Labor Supply Model," *Journal of Labor Economics*, 2, 550-566.

——— (1986): "The Econometrics of Piecewise-Linear Budget Constraints," *Journal of Business and Economic Statistics*, 4, 317-328.

——— (1990): "The Econometrics of Kinked Budget Constrains," *Journal of Economic Perspectives*, 4, 119-139.

Mroz, Thosmas A. (1987): "The Sensitivity of an Empirical Model of Marred Women's Hours of Work to Economic and Statistical Assumptions," *Econometrica*, 55, 765-799.

Owen, John D. (1978), "Why Part-Time Workers Tend to be in Low-Wage Jobs," *Monthly Labor Review*, 101, 11-14.

SAS Institute (1997): *SAS/OR Technical Report: The NLP Procedure*, Cary, NC: SAS Institute Inc.

Simpson, Wayne (1986): "Analysis of Part-Time Pay in Canada," *Canadian Journal of Economics*, 19, 798-807.

Stern, Nicholas (1986): "On the Specification of Labour Supply Functions," in *Unemployment, Search and Labour Supply*, ed. by Richard Blundell and Ian Walker. Cambridge: Cambridge University Press, 143-189.

Tummers, Martijn and Isolde Woittiez (1991): "A Simultaneous Wage and Labor Supply Model With Hours Restrictions," *Journal of Human Resources*, 26, 393-423.

Tunali, Insan (1986): "A General Structure of Models of Double-Selection and an Application to a Joint Migration/log-wages Process with Remigration," *Research in Labor Economics*, 8(B), 235-283.

Wales, T. J. and A. D. Woodland (1980): "Sample Selectivity and the Estimation of Labor Supply Functions," *International Economic Review*, 21, 437-468.

Zabel, Jeffrey E. (1993): "The Relationship between Hours of Work and Labor Force Participation in Four Models of Labor Supply Behavior," *Journal of Labor Economics*, 11, 387-416.

Table 1: Labor Force Status

<table>
<tr><th colspan="3" rowspan="2"></th><th rowspan="2">$W_r \leq W_p$</th><th colspan="2">$W_p < W_r \leq W_f$</th><th rowspan="2">$W_r > W_f$</th></tr>
<tr><th>$\mathbf{I}_2 \leq 0$</th><th>$\mathbf{I}_2 > 0$</th></tr>
<tr><td colspan="3">$H_f^* > \overline{H}$</td><td>Full 1</td><td>Full 2</td><td>—</td><td>—</td></tr>
<tr><td rowspan="4">$0 < H_f^* \leq \overline{H}$</td><td colspan="2">$H_p^* > \overline{H}$</td><td>Kink 1</td><td>—</td><td>—</td><td>—</td></tr>
<tr><td rowspan="2">$0 < H_p^* \leq \overline{H}$</td><td>$\mathbf{I}_1 \leq 0$</td><td>Kink 2</td><td>—</td><td>—</td><td>—</td></tr>
<tr><td>$\mathbf{I}_1 > 0$</td><td>Part</td><td>—</td><td>—</td><td>—</td></tr>
<tr><td colspan="2">$H_p^* \leq 0$</td><td>—</td><td>Kink 3</td><td>NLF 1</td><td>—</td></tr>
<tr><td>$H_f^* \leq 0$</td><td colspan="2">$H_p^* \leq 0$</td><td>—</td><td>—</td><td>—</td><td>NLF 2</td></tr>
</table>

Table 2: Variables Used for Study

Variables	Definition and Note
Age	Aged 25 – 60 years.
$Age^2/100$	Age squared in hundreds.
Education	Number of years of education.
Race	White = 1, Non-White = 0.
Marriage	Married = 1, Single = 0. Married but spouse absent is treated as single.
MSA	Metropolitan statistical areas = 1, Else = 0.
Midwest	Midwest region = 1, Else = 0.
South	South region = 1, Else = 0.
West	West region = 1, Else = 0.
Northeast	Northeast Region = 1, Else = 0. Northeast region is reference region.
Children < 6	Number of children under aged 6.
Children 6-18	Number of children aged 6 – 18.
Family Size	Number of family member.
Non-Labor Inc.	Sum of last year's survivor's income, interest income, dividends income, rent income, child support payment, alimony in thousands. If married, husband's last year earnings are added.
Wages	Hourly wage rate = usual weekly earnings / usual weekly hours of work.
Hours	Usual weekly hours of work.

Table 3: Mean Characteristics of the Sample

	Whole	FT	PT	LF	NLF
Age	39.60	39.92	39.79	39.90	38.36
	(9.36)	(9.43)	(8.92)	(9.34)	(9.37)
Education	13.22	13.55	13.07	13.46	12.20
	(2.63)	(2.50)	(2.32)	(2.48)	(2.95)
Race	0.81	0.81	0.87	0.82	0.76
	(0.39)	(0.39)	(0.33)	(0.38)	(0.43)
Marriage	0.58	0.52	0.72	0.55	0.67
	(0.49)	(0.50)	(0.45)	(0.50)	(0.47)
MSA	0.76	0.76	0.72	0.75	0.81
	(0.43)	(0.43)	(0.45)	(0.43)	(0.39)
Midwest	0.24	0.24	0.28	0.25	0.19
	(0.43)	(0.43)	(0.45)	(0.43)	(0.40)
South	0.31	0.33	0.24	0.31	0.31
	(0.46)	(0.47)	(0.43)	(0.46)	(0.46)
West	0.19	0.19	0.17	0.19	0.21
	(0.39)	(0.39)	(0.38)	(0.39)	(0.41)
Children $<$ 6	0.30	0.20	0.35	0.23	0.60
	(0.62)	(0.49)	(0.64)	(0.52)	(0.85)
Children 6-18	0.64	0.52	0.90	0.59	0.83
	(0.93)	(0.85)	(1.04)	(0.90)	(1.04)
Family Size	2.96	2.69	3.36	2.81	3.56
	(1.47)	(1.39)	(1.36)	(1.41)	(1.55)
Non-Labor Inc.	22.94	19.63	29.83	21.51	28.81
	(26.57)	(24.12)	(28.21)	(25.23)	(30.79)
(Excluding Hus-	1.04	1.10	1.07	1.09	0.85
band Wage)	(4.35)	(4.23)	(3.42)	(4.09)	(5.29)
Wages		12.21	9.56	11.72	
		(6.09)	(5.62)	(6.09)	
Hours		41.04	23.39	37.78	
		(4.59)	(6.88)	(8.54)	
Sample size	4674	3064	694	3758	916

[a] Standard deviations are reported in parentheses.
[b] Whole, FT, PT, LF, and NLF represent the whole sample, full-time workers, part-time workers, women in the labor force, and women not in labor force, respectively.

Table 4: Wages and Labor Supply: Discontinuous Budget Line Model

	OLS		DBL		Conditional DBL	
Wages						
Constant	-20.30^{**}	(1.67)	-18.48^{**}	(0.82)	-20.91^{**}	(1.63)
Age	0.68^{**}	(0.08)	0.72^{**}	(0.04)	0.69^{**}	(0.08)
$\text{Age}^2/100$	-0.75^{**}	(0.10)	-0.82^{**}	(0.04)	-0.77^{**}	(0.09)
Education	1.14^{**}	(0.03)	0.80^{**}	(0.03)	1.12^{**}	(0.05)
Race	0.36	(0.22)	0.56^{*}	(0.22)	0.45	(0.23)
MSA	1.38^{**}	(0.19)	0.60^{**}	(0.23)	1.25^{**}	(0.18)
Midwest	-1.43^{**}	(0.24)	-0.39^{*}	(0.18)	-1.08^{**}	(0.25)
South	-1.45^{**}	(0.23)	-0.55^{**}	(0.18)	-0.94^{**}	(0.26)
West	-0.41	(0.26)	0.22	(0.20)	-0.04	(0.26)
Full-Time Premium	2.22^{**}	(0.22)	4.58^{**}	(0.16)	2.83^{**}	(0.52)
Hours						
Constant	39.22^{**}	(2.69)	24.85^{**}	(4.89)	39.09^{**}	(3.34)
Age	-0.05	(0.14)	-1.17^{**}	(0.32)	-0.28	(0.19)
$\text{Age}^2/100$	-0.0005	(0.17)	1.28^{**}	(0.37)	0.25	(0.23)
Race	-0.32	(0.36)	1.09	(0.56)	-0.40	(0.39)
Marriage	-0.29	(0.41)	-0.69	(0.78)	-1.74^{**}	(0.36)
Children < 6	-2.14^{**}	(0.30)	-4.58^{**}	(0.34)	-2.17^{**}	(0.40)
Children 6-18	-1.47^{**}	(0.21)	-1.25^{**}	(0.33)	-1.59^{**}	(0.25)
Family Size	-0.08	(0.15)	-0.19	(0.23)	-0.14	(0.16)
Wages	0.29^{**}	(0.02)	2.27^{**}	(0.28)	0.60^{**}	(0.08)
Non-Labor Inc.	-0.04^{**}	(0.01)	-0.06^{**}	(0.01)	0.002	(0.005)
σ_ϵ			18.20^{**}	(1.19)	8.15^{**}	(0.18)
σ_e			7.06^{**}	(0.12)	3.83^{**}	(0.07)
σ_ν			5.31^{**}	(0.10)	5.14^{**}	(0.09)
$\rho_{\epsilon\nu}$			-0.74^{**}	(0.06)	-0.38	(0.08)
Log-likelihood			–27069.04		-24331.11	
Sample Size	3758		4674		3758	

[a] Standard errors are reported in parentheses.

[b] ** and * mean statistically significant at 1% and 5% respectively.

Table 5: Full-time Wage Premium

	Premium	
OLS	2.22**	(0.22)
Heckman's two-step method models		
a. probit selection (full-, part-time)	−1.01	(1.01)
b. ordered probit selection		
(full-, part-time, non-working)	0.42	(0.50)
c. bivariate probit selection		
(participation or not, full- vs. part-time)	−0.29	(1.56)
Discontinuous Budget Line models		
a. DBL	4.58**	(0.16)
b. Conditional DBL	2.83**	(0.52)

[a] Standard errors are reported in parentheses.

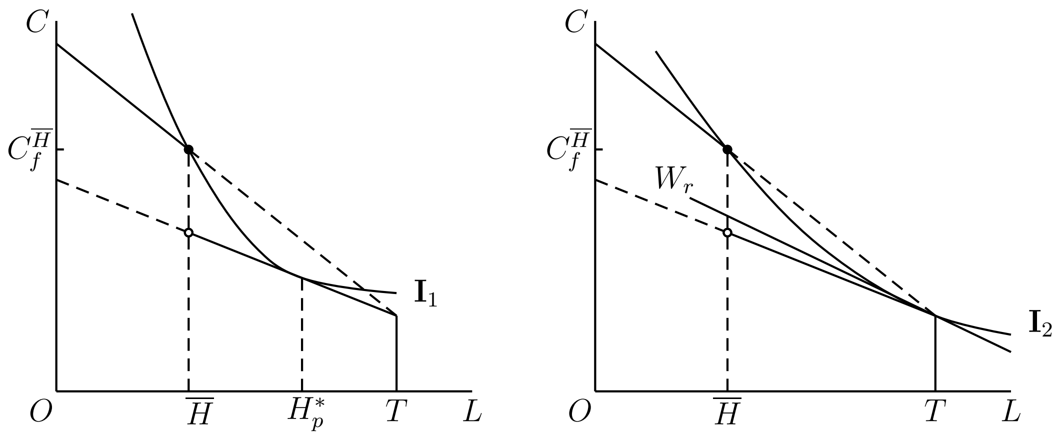
[b] ** and * mean statistically significant at 1% and 5% respectively.

Table 6: Elasticities

		Wage Elasticity		Income
		Compensated	Uncompensated	Elasticity
OLS	LF	0.511	0.097	–0.415
	FT	0.520	0.087	–0.432
	PT	0.475	0.137	–0.338
Tobit	LF	2.478	0.437	–2.042
	FT	2.522	0.395	–2.127
	PT	2.285	0.620	–1.666
Conditional Tobit	LF	0.569	0.124	–0.445
	FT	0.576	0.112	–0.464
	PT	0.539	0.175	–0.363
DBL	LF	1.464	0.749	–0.716
	FT	1.422	0.677	–0.745
	PT	1.645	1.061	–0.584
Conditional DBL	LF	0.171	0.198	0.026
	FT	0.151	0.179	0.027
	PT	0.259	0.280	0.022

[a] Compensated wage elasticity is $\alpha \cdot W/H - \delta W$, that is, uncompensated wage elasticity minus total income elasticity.
[b] The elasticity is the average of the elasticity of each woman.
[c] LF, FT, and PT represent women in the labor force, full-time workers, and part-time workers, respectively.
[d] The elasticity of FT (full-time workers) and PT (part-time workers) are computed using the wages and hours of workers who report working more than or equal to 35 hours, and less than 35 hours, respectively.

Figure 1: Indices for Utility Comparison with a Discontinuous Budget Line



(A) Index 1 (B) Index 2

Appendix: Specification of the Likelihood Function for the DBL Model

The errors (ϵ, ν, and e) are assumed joint normal with mean zero and the following covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma_\epsilon^2 & \sigma_{\epsilon\nu} & 0 \\ & \sigma_\nu^2 & 0 \\ & & \sigma_e^2 \end{bmatrix}.$$

Definitions

1. Errors:

   $\nu = W - \widehat{W}_i$, where $i = p, f$ when workers are observed as part- and full-time workers, respectively.

   $\eta = \alpha\nu + \epsilon$.

   $\xi_i = \epsilon + e = H - \widehat{H}_i - \alpha\nu$, where $i = p, f$ according to the assumed wage rate to derive the optimal hours of work.

   $\zeta_i = \eta + e = H - \widehat{H}_i$, where $i = p, f$ according to the assumed wage rate to derive the optimal hours of work.

2. Basic variances and correlation coefficients:

   $\rho = \frac{\sigma_{\epsilon\nu}}{\sigma_\epsilon \sigma_\nu}$.

   $\sigma_\eta^2 = \alpha^2\sigma_\nu^2 + \sigma_\epsilon^2 + 2\alpha\rho\sigma_\epsilon\sigma_\nu$.

   $\sigma_\xi^2 = \sigma_\epsilon^2 + \sigma_e^2$.

   $\sigma_\zeta^2 = \sigma_\eta^2 + \sigma_e^2$.

   $\rho_{\eta\zeta} = \sigma_\eta^2$.

3. Conditional means:

$\mu_{\xi|\nu} = \mu_{\epsilon|\nu e} = \rho\frac{\sigma_\epsilon}{\sigma_\nu}\nu.$

$\mu_{\epsilon|\nu\xi_i} = \frac{\sigma_\epsilon^2\sigma_\nu(1-\rho^2)\xi_i+\rho\sigma_\epsilon\sigma_e^2\nu}{\sigma_\nu(\sigma_\epsilon^2(1-\rho^2)+\sigma_e^2)}.$

4. Conditional variances:

$\sigma^2_{e|\nu} = \sigma_e^2.$

$\sigma^2_{\xi|\nu} = \sigma_\epsilon^2(1-\rho^2) + \sigma_e^2.$

$\sigma^2_{\epsilon|\nu e} = \sigma_\epsilon^2(1-\rho^2).$

$\sigma^2_{\epsilon|\nu\xi} = \sigma_\epsilon^2\left(1 - \frac{\sigma_\epsilon^2(1-\rho^2)+\rho^2\sigma_e^2}{\sigma_\epsilon^2(1-\rho^2)+\sigma_e^2}\right).$

Likelihood Function of the DBL Model

The likelihood function for the whole sample is

$$L = \prod_{H>0} Pr(H, W|H > 0) \cdot Pr(H > 0) \prod_{H=0} Pr(H = 0),$$

and the likelihood function for only the working sample is

$$L = \prod_{H>0} Pr(H, W|H > 0),$$

where $Pr(H, W|H > 0) = Pr(H, W)/Pr(H > 0)$ since $Pr(H, W, H > 0) = Pr(H, W)$, and $Pr(H, W|H > 0) \cdot Pr(H > 0) = Pr(H, W)$; $Pr(H, W)$, $Pr(H > 0)$, and $Pr(H = 0)$ are, respectively,

$$
\begin{aligned}
&Pr(H,W)\\
=\;& \left(\left[1-\Phi\left(\frac{\overline{H}-\widehat{H_f}-\alpha\nu-\mu_{\epsilon|\nu\xi_f}}{\sigma_{\epsilon|\nu\xi}}\right)\right]\cdot\frac{\phi\left(\frac{\xi_f-\mu_{\xi|\nu}}{\sigma_{\xi|\nu}}\right)}{\sigma_{\xi|\nu}}\cdot\frac{\phi\left(\frac{\nu}{\sigma_\nu}\right)}{\sigma_\nu}\right)\cdots(a)\\
+\;& \left(\begin{array}{c}\left[\Phi\left(\frac{\min\left(\overline{H}-\widehat{H_f},LR\right)-\alpha\nu-\mu_{\epsilon|\nu\xi_p}}{\sigma_{\epsilon|\nu\xi}}\right)-\Phi\left(\frac{-\widehat{H_p}-\alpha\nu-\mu_{\epsilon|\nu\xi_p}}{\sigma_{\epsilon|\nu\xi}}\right)\right]\cdot\\ \frac{\phi\left(\frac{\xi_p-\mu_{\xi|\nu}}{\sigma_{\xi|\nu}}\right)}{\sigma_{\xi|\nu}}\cdot\frac{\phi\left(\frac{\nu}{\sigma_\nu}\right)}{\sigma_\nu}\end{array}\right)\cdots(b)\\
+\;& \left(\begin{array}{c}\left[\Phi\left(\frac{\overline{H}-\widehat{H_f}-\alpha\nu-\mu_{\epsilon|\nu e}}{\sigma_{\epsilon|\nu e}}\right)-\Phi\left(\frac{\max\left(-\widehat{H_p},LR,\frac{\mathcal{D}}{\mathcal{E}}\right)-\alpha\nu-\mu_{\epsilon|\nu e}}{\sigma_{\epsilon|\nu e}}\right)\right]\cdot\\ \frac{\phi\left(\frac{H-\overline{H}}{\sigma_e}\right)}{\sigma_e}\cdot\frac{\phi\left(\frac{\nu}{\sigma_\nu}\right)}{\sigma_\nu}\end{array}\right)\cdots(c)\\
+\;& \left(\begin{array}{c}\left[\Phi\left(\frac{\min\left(\overline{H}-\widehat{H_f},-\widehat{H_p}\right)-\alpha\nu-\mu_{\epsilon|\nu e}}{\sigma_{\epsilon|\nu e}}\right)-\Phi\left(\frac{\max\left(-\widehat{H_f},\frac{\mathcal{D}}{\mathcal{E}}\right)-\alpha\nu-\mu_{\epsilon|\nu e}}{\sigma_{\epsilon|\nu e}}\right)\right]\cdot\\ \frac{\phi\left(\frac{H-\overline{H}}{\sigma_e}\right)}{\sigma_e}\cdot\frac{\phi\left(\frac{\nu}{\sigma_\nu}\right)}{\sigma_\nu}\end{array}\right)\cdots(d),
\end{aligned}
$$

$$
\begin{aligned}
&Pr(H>0)\\
=\;& \Psi\left(\frac{\widehat{H_f}}{\sigma_\zeta},\ \frac{\widehat{H_f}-\overline{H}}{\sigma_\eta},\ \rho_{\eta\zeta}\right)\cdots(a')\\
+\;& \left[\Psi\left(\frac{\widehat{H_p}}{\sigma_\zeta},\ \frac{\min\left(\overline{H}-\widehat{H_f},LR\right)}{\sigma_\eta},\ -\rho_{\eta\zeta}\right)-\Psi\left(\frac{\widehat{H_p}}{\sigma_\zeta},\ -\frac{\widehat{H_p}}{\sigma_\eta},\ -\rho_{\eta\zeta}\right)\right]\cdots(b')\\
+\;& \Phi\left(\frac{\overline{H}}{\sigma_e}\right)\cdot\left[\Phi\left(\frac{\overline{H}-\widehat{H_f}}{\sigma_\eta}\right)-\Phi\left(\frac{\max\left(-\widehat{H_p},LR,\frac{\mathcal{D}}{\mathcal{E}}\right)}{\sigma_\eta}\right)\right]\cdots(c')\\
+\;& \Phi\left(\frac{\overline{H}}{\sigma_e}\right)\cdot\left[\Phi\left(\frac{\min\left(\overline{H}-\widehat{H_f},-\widehat{H_p}\right)}{\sigma_\eta}\right)-\Phi\left(\frac{\max\left(-\widehat{H_f},\frac{\mathcal{D}}{\mathcal{E}}\right)}{\sigma_\eta}\right)\right]\cdots(d'),
\end{aligned}
$$

and

$$
\begin{aligned}
& Pr(H=0) \\
= \;& \Psi\left(-\frac{\widehat{H_f}}{\sigma_\zeta},\ \frac{\widehat{H_f}-\overline{H}}{\sigma_\eta},\ -\rho_{\eta\zeta}\right)\cdots(a'') \\
+ \;& \left[\Psi\left(-\frac{\widehat{H_p}}{\sigma_\zeta},\ \frac{\min\left(\overline{H}-\widehat{H_f}, LR\right)}{\sigma_\eta},\ \rho_{\eta\zeta}\right)-\Psi\left(-\frac{\widehat{H_p}}{\sigma_\zeta},\ -\frac{\widehat{H_p}}{\sigma_\eta},\ \rho_{\eta\zeta}\right)\right]\cdots(b'') \\
+ \;& \Phi\left(-\frac{\overline{H}}{\sigma_e}\right)\cdot\left[\Phi\left(\frac{\overline{H}-\widehat{H_f}}{\sigma_\eta}\right)-\Phi\left(\frac{\max\left(-\widehat{H_p}, LR, \frac{\mathcal{D}}{\mathcal{E}}\right)}{\sigma_\eta}\right)\right]\cdots(c'') \\
+ \;& \Phi\left(-\frac{\overline{H}}{\sigma_e}\right)\cdot\left[\Phi\left(\frac{\min\left(\overline{H}-\widehat{H_f}, -\widehat{H_p}\right)}{\sigma_\eta}\right)-\Phi\left(\frac{\max\left(-\widehat{H_f}, \frac{\mathcal{D}}{\mathcal{E}}\right)}{\sigma_\eta}\right)\right]\cdots(d'') \\
+ \;& \Phi\left(\frac{\min\left(\frac{\mathcal{D}}{\mathcal{E}},\ -\widehat{H_p}\right)}{\sigma_\eta}\right)\cdots(e''),
\end{aligned}
$$

where ϕ and Φ are the standard normal density and distribution function, respectively, and Ψ is the standard bivariate normal distribution function; $\widehat{W_i}$ means $\widehat{W_f}$ ($\widehat{W_p}$) if the workers are observed as full-time (part-time) workers.

(a), (a'), and (a'') represent probabilities when the preferred choice is working full-time; (b), (b'), and (b'') represent probabilities when the preferred choice is working part-time; (c), (c'), and (c'') represent probabilities when the preferred choice is working at kink point ($\overline{H}_{\text{Kink A}}$); (d), (d'), and (d'') represent probabilities when the preferred choice is working at kink point ($\overline{H}_{\text{Kink B}}$); (e'') represents probability of choosing non-working because the preferred choice is not-working.