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EEENS 6340/4340 The Earth

Homework I: Gravity and Gravity Anomalies

1. Assume that the Earth is a perfect sphere with a radius, r, of 6,300 km and that acceleration due to gravity, g, on the surface of the Earth is 9.81 m/s^2. The gravitational constant, G, is 6.67 x 10^{-11} m^3/kgs^2. Given that:

$$g = \frac{G\,M_E}{r^2}$$

   calculate M_E, the mass of the Earth.

2. Calculate the mean density of the Earth based on your answer to question 1.

3. The gravitational force of the Earth acting on a satellite is $GM_E m/r^2$, where m is the mass of the satellite and r is the distance between the centers of mass of the two bodies. For a satellite in a circular orbit around the Earth this force is balanced by the outward centrifugal force acting on the satellite, $m\omega^2 r$, where ω is the angular velocity of the satellite. The angular velocity can be related to the period, T, (time necessary to complete one revolution) by

$$T = 2\pi/\omega$$

   From this information derive an equation that relates the period, T, to orbital radius, r, and the mass of the Earth, M_E.

4. Given that the moon has a period of 27.3 days, and the mass of the Earth M_E is 5.98 x 10^{24} kg, calculate the orbital radius of the moon using the equation derived in problem 3.

5. A planet has the same mean density as the Earth, but has a radius 2 times as large as the Earth.
   a. If a woman has a mass of 50 kg on the Earth, what is her mass on this planet?
   b. What is the value of g on the surface of this planet?
   c. How much would the woman weigh on this planet?
   d. What velocity would be required for a satellite to orbit this planet just above the its surface? (On Earth, the satellite would have to have a velocity of about 8000 m/s in order to orbit just above the surface.)

6. Measurements of gravity at 5 stations in the U.S. are as follows

	Cambridge, MA	Mead Ranch, KS	Yosemite, CA	Mt. Whitney, CA	Pikes Peak, CO
Elevation, h, (m asl)	14.0	1005.0	2555.0	4418.4	4293.4
Observed gravity, g_m, (mgals)	980403.00	979760.00	979269.00	978735.00	978959.00
Standard gravity, g_o, at latitude of station (mgals)	980407.00	980084.00	980041.00	979881.00	980081.00

a. For each station calculate the free-air gravity anomaly, Δg_F, and simple Bouguer anomaly, Δg_B, assuming a uniform density of the crust, $\rho = 2670$ kg/m^3.

The free-air anomaly is given by:

$$\Delta g_F = g_m + h(0.3086 \text{ mgals/m}) - g_o$$

The Bougeur anomaly is given by:

$$\Delta g_B = \Delta g_F - 2\pi G\rho h + TC$$

Where G is the gravitational constant, 6.67 x 10^{-11} m^3/kgs^2, ρ is the average crustal density, and h is the elevation of the station. A topographic correction, TC, should be applied to the stations at Pikes Peak (+57 mgals) and Mt. Whitney (+76 mgals) because of the extreme relief in these areas. Make sure your units are consistent (1 Gal = 10^{-2}m/s^2, 1 mgal = 10^{-3} Gal).

b. Compare the Bouguer anomalies at the five stations by plotting them versus elevation. What differences do you notice between the first three stations and the other two? How do these differences relate to differences in the free-air anomalies?

c. In order to interpret the differences we will first consider a simple Isostatic correction to the gravity anomalies. Suppose in a simple minded way that the root of the mountains is a flat infinite plate (as in the Bouguer correction) of thickness H. What must be the relationship between h, the elevation, and H, the thickness of the root, in order for perfect compensation to take place and reduce the isostatic anomaly to 0? Hint: the isostatic anomaly, Δg_I, would be defined as:

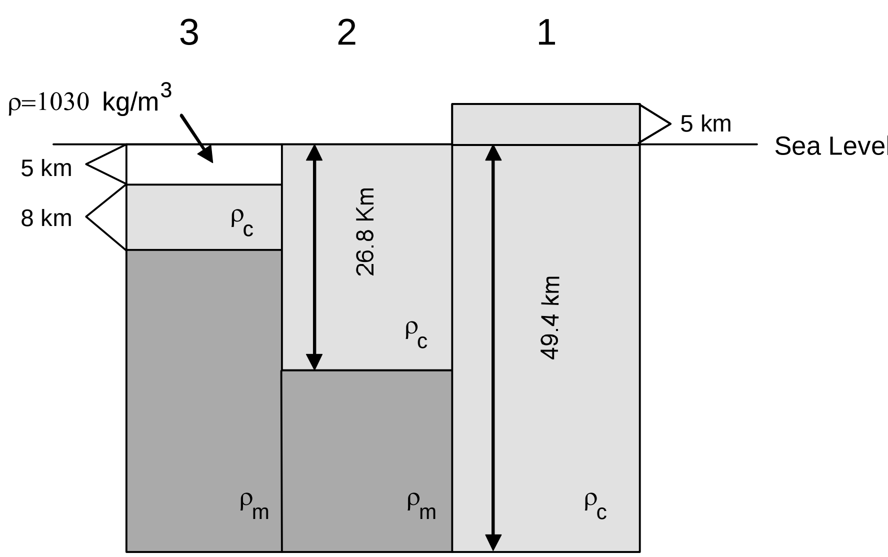
$$\Delta g_I = \Delta g_B + 2\pi G\rho H$$

d. Show that using this simple minded correction, and assuming a topographic correction of 0, that the free-air and isostatic anomalies are equal.

e. Note that the Yosemite and Mt. Whitney stations are both in the Sierra Nevada Mountains of California. If we assume the simple minded isostatic correction in d., and that the Yosemite station has the average elevation of the Sierra Nevada, does this suggest that mountain ranges are compensated peak by peak (i.e. each peak has its own root) or that mountain ranges are compensated on a regional basis with a crustal thickness corresponding to the range's average height?

f. An isostatic anomaly measures the departure from isostatic equilibrium. Where the isostatic anomaly is large and positive, the area must sink to restore isostatic equilibrium; where the isostatic anomaly is large and negative, the region must rise. What kind of geologic evidence would we look for to determine whether an area is rising or sinking?

7. In the diagram below, section 1 represents a high plateau with a mean elevation of 5 km, section 2 represents a continental region at sea level, and section 3 is a section of the deep ocean. Thicknesses of individual units have been measured seismically. Assume that all sections are in isostatic equilibrium and calculate the densities ρ_c and ρ_m of the crust and mantle respectively.



8. Imagine that in the diagram above, the mountain range in section 1 is rapidly eroded so as to reduce its elevation to 3 km above sea level. Now imagine that isostatic compensation is restored to section 1. What will be the elevation of the mountain range in section 1 after isostatic compensation has been restored?