Repeated measures multiple regression.

For the equivalent of SxA and S/AxB see Cohen & Cohen

Gully (1994) adapts Cohen & Cohen to allow for continuous predictors.

What is below is my notation and extractions, using one between-subjects predictor (continuous or categorical), one within predictor, and the interaction term. The example carried throughout has 40 subjects, 1 continuous predictor (between), three levels of a within, and the interaction. (The data are Seth Kaplan's).

Skip to page 8 for the quick-and-dirty method using GLM

1. Ascertain what proportion of the variance is between subjects.
   a. Find each subjects average score on the within variable.
      e.g., if you have 40 subjects with scores on each 3 trials, you have 40 average scores.
   b. Find the population variance of these scores. Note: SPSS and Excel return the sample variance, so you need to convert (i.e., multiply by N-1 then divide by N).
      e.g., Imagine the sample variance was .512 That would make the population variance .512(39)/40 = .4992
   c. Place all 120 scores in a single array, and find their population variance.
      e.g., If the sample variance is .89 the population variance is .8826
   d. The ratio of 1b/1c is R^2 between subjects
      e.g., .4492/.8826 = .5656
   e. df between subjects is the number of participants-1
      e.g., 40-1

2. Ascertain what proportion of the variance is within subjects. 1 - R^2 between
      e.g., 1-.5656
   a. df within subjects is the number of scores - dfbetween - 1
      e.g., 120-39-1 = 80

3. Effect code the within subjects variable with g-1 vectors.
      e.g., in our example, that gives us 3-1 vectors for the within
      Note that eats 2 df, which is what would be used in ANOVA

4. Center the continuous variable, which is a single vector eating 1 df

5. The interaction is estimated with the products of 3 and 4.
      e.g., in our example, you'll have 2 vectors, each created by the product of the continuous variable and one of the within vectors

6. Conduct a regression analysis using the within vectors in the first step, the continuous variable in the second step, and the interaction vectors in the third step.
Save each ΔR^2 from the print-out. The significance tests are wrong, because they are not using the correct error terms and df.
Also save the overall R^2 for each equation.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.088[a]	.008	-.009	.9476	.008	.455	2	117	.636
2	.382[b]	.146	.124	.8828	.138	18.801	1	116	.000
3	.425[c]	.181	.145	.8721	.035	2.428	2	114	.093

a. Predictors: (Constant), V2, V1
b. Predictors: (Constant), V2, V1, SESCENT
c. Predictors: (Constant), V2, V1, SESCENT, V2SES, V1SES

7. Conduct a regression analysis using the within factor vectors and the interaction vectors. What you need to save here is the R^2 for the equation, which represents the systematic variance accounted for by within subjects terms. The total within minus this value is the error within

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.088[a]	.008	-.009	.9476	.008	.455	2	117	.636
2	.206[b]	.043	.009	.9388	.035	2.096	2	115	.128

a. Predictors: (Constant), V2, V1
b. Predictors: (Constant), V2, V1, V2SES, V1SES

8. Note that if you had additional between predictors, you'd have to do the equivalent of #7 with the between....ie find the R^2 for the equation for all the systematic between variance and use that as "R^2 for only between predictors" in 9b

9. Conduct the hierarchical regression analyses as follows:

a. Trials (V1 + V2)

$$\frac{\Delta R^2 \text{for trials (from \#6)}}{R^2\text{within} - R^2\text{only and all within preds}} \times \frac{\text{N - dfbetween - all within predictors - 1}}{\text{number of predictors for trials}}$$

e.g.,

$$\frac{.008}{.4344 - .043} \times \frac{120 - 39 - 4 - 1}{2}$$

b. Between continuous predictor

$$\frac{\Delta R^2 \text{for between vector (from \#6)}}{R^2\text{between} - R^2 \text{ only between preds}} \times \frac{\text{N - dfwithin - all between predictors - 1}}{\text{\# of between predictors in this step}}$$

$$\frac{.138}{.5656 - .138} \times \frac{120 - 80 - 1 - 1}{1}$$

c. trials-by-continuous interaction

$$\frac{\Delta R^2 \text{for txc (from \#6)}}{R^2\text{between} - R^2 \text{ only between terms}} \times \frac{\text{N - dfbetween - all within predictors - 1}}{\text{\# of between predictors in this step}}$$

$$\frac{.035}{.4344 - .043} \times \frac{120 - 39 - 4 - 1}{2}$$

9. Janet's simplified interpretation.

The equations above essentially are from Gully 1994, using my notation. A few things worth noting may make the whole thing easier to think about, though I don't have the statistical savvy to do the proofs.

First, for any problems that I've worked out, the right-hand terms are equivalent to the df for the F-ratio of that term. That would make sense, insofar as the usual formula for ΔR^2 involves dividing the numerator and denominator of each of the left-hand terms by its df. (Which is the same thing as multiplying by the reciprocal).

i.e.,

c. trials-by-continuous interaction could also be written as

$$\frac{(\Delta R^2 \text{for txc})/df_{num}}{(R^2\text{between - } R^2 \text{ only between terms})/df_{denom}}$$

Where the df $_{num}$ for ΔR^2 are the number of predictors added in this step and
df $_{num}$ are for the error term dfwithin - df#predictors - 1

$$\frac{.035 / 2}{(.4344 - .043)/ 76}$$

Second, one notices that the denominator is just what you'd expect from an error term, the unaccounted for within variance. It is the proportion of variance that is within subjects with any systematic variance accounted for by *all within predictors* subtracted out (i.e., the unaccounted for within variance). The between denominator follows the same logic. All of this may be written down in some chapter or article, but I must have missed it.

Finally, steps 7 and 8 really aren't necessary: You can get this by subtraction
e.g., R^2within - R^2trials - R^2interaction

10. Example Table

Step	Predictors	R^2 for equation	ΔR^2	$F\Delta$	df
1.	Trial	.008	.008	0.777	2,76
2.	Continuous Between	.146	.138	12.26**	1,38
3	TxC	.181	.035	3.40*	2,76

Note: Within variables (Trials and TxC) account for .043 of the total variance
Between variable (continuous) accounts for .138 of the total variance

11. If desired, you can show the increment in the variance within (or between) accounted for by adding the variable. e.g., .035/.4344 = .0806 is the increment in the within variance that is accounted for by the interaction term.

12. Finally, you can graph your interaction. The bs that you have in the final equation are right, even if their significance tests make no sense (remember: the wrong error term is used)... I know it looks weird to have multiple bs for one effect (e.g., there are 2 bs for the trial effect), but that's okay (Schmelkin and Pedhauzer comment on this point when you are using effect coding). Use that equation to draw your picture. For example, if the final equation were

predY = 3.367 + -.117V1 + .05833V2 + .03973S + -.01194V1S + .0281V2S

Rewrite the equation as

(.03973 + -.01194V1 + .0281V2)S + (3.367 + -.117V1 + .05833V2)

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	3.367	.086		38.921	.000
	V1	-.117	.122	-.101	-.954	.342
	V2	5.833E-02	.122	.051	.477	.634
2	(Constant)	3.367	.081		41.777	.000
	V1	-.117	.114	-.101	-1.024	.308
	V2	5.833E-02	.114	.051	.512	.610
	SESCENT	3.973E-02	.009	.372	4.336	.000
3	(Constant)	3.367	.080		42.289	.000
	V1	-.117	.113	-.101	-1.036	.302
	V2	5.833E-02	.113	.051	.518	.605
	SESCENT	3.973E-02	.009	.372	4.389	.000
	V1SES	-1.194E-02	.013	-.091	-.932	.353
	V2SES	2.810E-02	.013	.215	2.196	.030

a. Dependent Variable: SCORE

Create 3 simple regression equations for each level of the within variable

(.03973 + -.01194V1 + .0281V2)S + (3.367 + -.117V1 + .05833V2)

Your effect coding for the within factor was:

V1 V2: -1 -1 (good odds)
V1 V2: 0 +1 (fair odds)
V1 V2: +1 0 (poor odds)

The standard deviation of the continuous predictor was 8.83

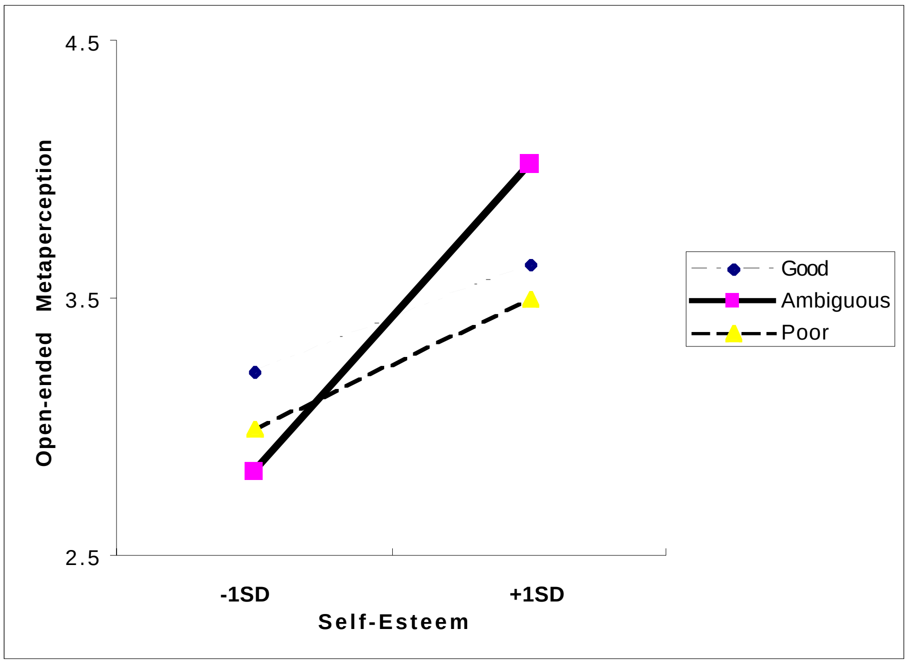
Good odds equation: (.03973 + -.01194(-1) + .0281(-1))S + (3.367 + -.117(-1) + .05833(-1))
reducing to: .02357S + 3.42567

Fair odds equation: (.03973 + -.01194(0) + .0281(+1))S + (3.367 + -.117(0) + .05833(+1))
reducing to: .06773S + 3.42533

Poor odds equation: (.03973 + -.01194(1) + .0281(0))S + (3.367 + -.117(+1) + .05833(0))
reducing to: .02779S + 3.25

Substituting -8.83 and +8.83, the 6 points are:

Good:	3.22	3.63
Fair:	2.83	4.02
Poor:	3.00	3.50



RMMR using the General Linear Model

1. In SPSS, select analysis with general linear model, repeated measures.
   a. enter the variables for your within levels as usual
   b. enter your continuous variable as a covariate
   c. select model, and build a custom model that produces interaction terms

2. F and df for ΔR^2 are given by the tested effects, assuming sphericity.

Tests of Between-Subjects Effects

Measure: MEASURE_1
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	1360.133	1	1360.133	1143.093	.000
SESCENT	14.652	1	14.652	12.314	.001
Error	45.215	38	1.190		

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
FACTOR1	Sphericity Assumed	.817	2	.408	.748	.477
	Greenhouse-Geisser	.817	1.792	.456	.748	.464
	Huynh-Feldt	.817	1.925	.424	.748	.472
	Lower-bound	.817	1.000	.817	.748	.393
FACTOR1 * SESCENT	Sphericity Assumed	3.694	2	1.847	3.383	.039
	Greenhouse-Geisser	3.694	1.792	2.061	3.383	.045
	Huynh-Feldt	3.694	1.925	1.919	3.383	.041
	Lower-bound	3.694	1.000	3.694	3.383	.074
Error(FACTOR1)	Sphericity Assumed	41.489	76	.546		
	Greenhouse-Geisser	41.489	68.102	.609		
	Huynh-Feldt	41.489	73.136	.567		
	Lower-bound	41.489	38.000	1.092		

3. To find the ΔR^2 for each of the steps, you need to consider that you are thinking about the proportion of the total variance that this step adds to the prediction of the criterion (i.e., the sums of squares for ΔR^2 relative to the total variability in the data.) To find the corrected total (i.e., corrected for the mean) add up the variance sources: the 3 effects and the 2 error terms:

$$14.652+45.215+.817+3.694+41.489 = 105.867$$

So, for example, ΔR^2 for the interaction is 3.694/105.867 = .035

4. R^2 for any given equation is the sum of what came before plus the new variance:

e.g., .008 + .138 + .035 = .181, the R^2 for the equation with all 3 predictors

5. To find the simple slopes (i.e., the slope at each level of the within factor), using simple regression

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.250	.168		19.298	.000
	SESCENT	2.779E-02	.019	.229	1.451	.155

a. Dependent Variable: NARPO

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.425	.113		30.210	.000
	SESCENT	6.783E-02	.013	.649	5.262	.000

a. Dependent Variable: NARNEU

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.425	.126		27.224	.000
	SESCENT	2.356E-02	.014	.258	1.647	.108

a. Dependent Variable: NARGO