# Choosing Mutually Orthogonal Coefficients

- 1. Select a comparison that tests the relations among all groups (i.e. Coefficients should carry the sign + or -. There should be no zeros as coefficients). It is handy if this comparison is the experimenter's hypothesized a priori comparison.
- 2. Make sure the sum of those coefficients is zero.
- 3. Make sure all the plus signs are adjacent to one another and all the minus signs are adjacent to one another.
- 4. Draw a line between the + and signs of the first contrast.
- 5. Choose to create your comparison on either side of the line (this means all your + and signs go on one side).
- 6. Create another set of coefficients that sum to zero. If you follow the rules carefully, they will be mutually orthogonal to the previous and subsequent comparisons.
- 7. Draw a line between the + and signs of the second contrast. Repeat steps 5 and 6 as necessary.
- 8. Never "cross a line" in a subsequent contrast. This means that the coefficients that have plus and minus signs must always be between the vertical lines you draw. You can't have a plus on one side of a vertical line, and a minus on the other.

An example with 5 groups

Note: To **prove** a set of coefficients is mutually orthogonal, sum the cross-products of each pair of coefficients. For example, in the immediately preceding example, C2 and C3 are proven orthogonal by showing that product of the coefficients for V are  $-1^*-1 = +1$ , for V are  $-1^*+1 = -1$ , for V are  $-1^*-1 = +1$ , for V are  $-1^*-1 = -1$ , for V are  $-1^$ 

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Test	Statistic	Formula	df	a priori?	Type	alpha
Orthogonal Comparison	F	<u>L²/nΣα²</u> MSwithin	1 dfw	yes	Σα=0 df=k	each .05
Bonferroni Comparison	F	<u>L²/nΣα²</u> MSwithin	1 dfw	yes	Σα≠0 df≠k	each .05/k
Scheffé Comparison		<u>nΣa²)/g-1</u> Swithin	g-1 dfw	no	∑a≠0 df≠k	family .05
Tukey	q √N	<u>M - M</u> ASwithin/n	g dfw	yes	all pairs	family .05
Newman-Keuls		<u>1 - M</u> NSwithin/n	g* dfw	yes	all pairs	family .05

 $g^*$  number of groups within the comparison range  $\mathcal{M}$  = the sample mean

# Which kind of ANOVA is it?

When designing an experiment, researchers choose among various types of experimental designs. Certain kinds of questions are best tested with between-subjects designs (i.e., one-way ANOVA or a between-subjects factorial design, such as a  $2 \times 2 \times 3$ ). Other questions require purely within-subjects designs (e.g.,  $S \times A$ ,  $S \times A \times B$ ) or a mixture of between and within factors (e.g.,  $S/A \times B$ ).

In making their decision about which design to use, researchers consider a variety of issues; these issues include theory, experimental paradigm, and state-of-the-art technology. We haven't considered these issues in Psych 212; you will probably consider them in your methodology courses. We have discussed some reasons for the choice of one design over another, however. For example, if there are likely to be carry-over effects from one level of an independent variable to another level (no matter how long the interval between levels), one might consider making that IV between-subjects. As another example, when the cost of data collection is high, a within-subjects or mixed design might be preferred.

It is not the purpose of this course to teach you how to create an experiment. But, once you are ready to create an experiment, you will have some tools with which to analyze data. For the purposes of this course, you should be able to recognize different experimental designs so that you can analyze them appropriately. Following the steps below will help you decide which ANOVA to use.

- 1. Identify the dependent variable (i.e., what is being measured). Now ask yourself: are subjects completing that measure more than once? If the answer is no, then you have a between subjects design. If the answer is yes, you have at least one within subjects variable.
- 2. Identify the independent variable(s). These are the factors that are responsible for systematic variability among subjects' responses to the DV (as opposed to error variability). Some Ivs will be manipulated by the researcher and as such are true causes. Other IVs will be "Subject Variables" which the researcher expects contribute variability but they aren't really manipulated (e.g., gender, IQ). (Remember that if you answered "yes" to question 1, subjects is a random independent variable).
- 3. For each of the independent variables (except for the random IV subjects), count the number of levels
- 4. Draw the design as a series of boxes.
- 5. For the first independent variable (except for the random IV subjects), ask yourself whether any given subject receives only one level of it or if a subject receives all levels of that IV. If you answer the former, then this IV is between subjects. On your drawing, write "between" in parenthesis next to the factor name. If you answer the latter, this IV is within subjects. On your drawing, write "within" in parenthesis next to the factor name.
- 6. Repeat step 5 for as many IVs as you have identified.
- 7. If all of your IVs are between, describe the design in terms of how many IVs there are (and the number of levels in each IV). For example, if there are three independent variables, each of which have 2 levels, you have a 2 X 2 X 2 between subjects design (in which case you do a 3-way between subjects ANOVA). For purposes of this course, you know 1-way, 2-way, and 3-way between-subjects ANOVAs. (There are 4-ways, 5-ways, etc).
- 7a. If all your IVs are within, you have an S by "something else" design. For purposes of this course, S by "something else" has been S X A and S X A X B. (There are S X A X B X C, in which there are three within-subjects variables, etc).
- 7b. For the purposes of this course, if one IV is within and the other is between, you have an  $S/A \times B$  design. As you might imagine, there are more complex designs like  $S/A \times S/B \times C$ , in which A and B are between subjects while C is within.

#### **Exam Review**

\*Need to know for all subsequent noncumulative examinations as well as current examination; the final is cumulative, so anything covered in the course is "fair game"

## Exam 1

#### Memorize or Distinguish among:

Variables and constants
Population, sample
Parameter, statistic
Mean,median, mode
range, standard deviation, variance
skew, kurtosis, modality
normal distribution characteristics
sample vs sampling distribution
sample vs sampling distribution
the Central Limit Theorem
Why N-1 for sample standard deviation
why expected value of F is 1 under the pull\*
similarities and differences among the similarities and differences among the sample of the pull\*

Why expected value of F is 1 under the null\* similarities and differences among the tests subsequent to ANOVA

Determinants of power independent and dependent variables\*
Type I and Type II errors assumptions of b/n groups ANOVA

determine which procedure is appropriate given described study\*

#### Know how to:

Recognize formulas\*

Compute degrees of freedom\*

Find percentile rank

Use statistical tables (e.g., z, F)\*

State the null and alternative hypotheses\*
determine orthogonality

"split up" variance in the various designs\*

compute 1-way b/n groups ANOVA using heuristic formula and raw score formulae
conduct orthogonal, scheffe, and bonferroni comparions
conduct tukey and neuman-keuls tests

compute standard deviation and variance for populations and samples
find confidence interval around the mean

Decide whether to retain or reject the null hypothesis\*
find N required for x% power in one-way ANOVA

## Memorize or Distinguish among:

compute chi-square tests

problems with unequal n, how to deal with fixed effect vs random effects independent variables assumptions of repeated measures ANOVA Box's Epsilon 2-way interaction

Know how to: interpret graphs\*

recognize simple and main effects\*

use Epsilon to adjust df

# Exam 2

why Ss are an IV in repeated measures designs between vs within independent variables problems with difference scores appropriate error terms in the repeated measures designs main effect simple effect

recognize 2-way interactions describe designs in terms of levels, type of IVs\*

# Exam 3

#### Memorize or Distinguish among:

assumptions of pearson product moment correlation what confidence intervals around rho means r as extent to which x and y have same absolute value percentile rank scatter plot equation for multiple r

#### Know how to:

Find a confidence interval around rho compute pearson r, slope, intercept, regression line test significance of unstandardized regression coefficients F-test for multiple interpret output of multiple regression analyses f

z-test for independent <u>r</u> s what <u>r</u> means regression line, slope, intercept standard vs hierarchical MR what R<sup>2</sup> means

interpret and test the significance of a r and R use a simple regression line find standard error of estimate F test for change in R-squared ind N required for x% power in simple regression

# Formulas

Population Variance:  $\frac{\sum x^2 - (\sum x)^2/N}{N}$ The Mean:  $\Sigma x/N$ 

z-score =  $(x-\mu)/\sigma$ 

t-test for sample:  $(M-\mu)/s/\sqrt{N}$ variance standard error

of the mean:  $\sigma^2/N$ vs. population

the mean:

s/√N

 $\phi' = \sqrt{\frac{[SSb/N]/[MSw]}{}}$ SSContrast= <u>L</u><sup>2</sup> Power: 1-β linear contrast: L=∑aT

 $\phi^2/\phi'^2 = n \text{ per group}$ eta for ncF  $oldsymbol{\phi}=oldsymbol{\phi}$ '  $\sqrt{n}$ 

CI around population mean:  $M \pm t(s/\sqrt{N})$ 

control for family-

Tukey/Newman-Keuls:  $(M_1 - M_2)/\sqrt{MSWithin/n}$ wise error rate: ∝/k

Chi-square:  $\Sigma (O-E)^2/E$ 

Heuristic F=  $\frac{nS_x^2}{\sum S^2/g}$ GrandTotal =  $\sum x$  $CF = GT^2/N$ 

One-way between groups Two-way between groups

 $\sum x^2$  - CF  $\sum x^2$  - CF  $\frac{\sum \text{Cells}^2/\text{n} - \text{CF}}{\sum \text{R}^2/\text{cn} - \text{CF}}$   $\frac{\sum \text{C}^2/\text{rn} - \text{CF}}{\sum \text{C}^2/\text{rn} - \text{CF}}$  $\Sigma T^2/n - CF$ SSTotal-SSBetween

BetweenCells-Row-Column

Total-BetweenCells

NOTE: M refers to the sample mean

SxA One-way Repeated Measures:

 $S/A \times B$ : Two-way mixed design

Total =  $\sum x^2 - CF$   $\sum S^2/a - CF$ Within = Total-Between  $\sum A^2/s - CF$  $\sum xa$  (error) = Within-A  $\begin{array}{l} \sum x^2 - (\sum x)^2/N \\ \sum S^2/b - (\sum x)^2/N \\ \sum A^2/bn - (\sum x)^2/N \\ \text{SSBetween-SSA} \\ \text{SSTotal-SSBetween} \\ \sum B^2/an - (\sum x)^2/N \\ \sum (AB)^2/n - (\sum x)^2/N - \text{SSB-SSA} \\ \text{SSWithin-SSB-SSAxB} \end{array}$ 

 $\frac{\Delta R^2 / k_{\Delta}}{[(1-R_o^2)/(N-k_o-1)]}$ 

regression line: y' = a + bx slope:  $\frac{N\sum xy - \sum x\sum y}{[N\sum x^2 - (\sum x)^2]}$  slope:  $r(S_y/S_x)$   $r^2/[(1-r^2)/(N-2)]$ 

CI around rho:  $Z' \pm z(1/\sqrt{N-3})$