

Some Basics of Bargaining Theory

These notes provide a very elementary introduction to bargaining theory, with an emphasis on the Nash cooperative solution. For those seeking a more systematic introduction, see: Ehud Kalai (1985) "Solutions to the Bargaining Problem" from L. Hurwicz, D. Schmeidler, and H. Sonnenschein, eds. *Social Goals and Social Organization*; Hervé Moulin (1988) *Axioms of Cooperative Decision Making* (Chapter 3); or Alvin Roth (1979) *Axiomatic Models of Bargaining*.

I. An Introduction to Bargaining Theory

A. What issues arise in bargaining

1. The **joint efficiency problem**: bargainers seek to identify a unique, mutually beneficial contract (or treaty). Thus, from the set of all feasible payoffs, they first need to identify the set of *undominated* payoffs.
2. The **payoff distribution problem**: once the jointly efficient outcomes have been identified, the bargainers must choose an element from this set.
3. The **strategy coordination problem**: the bargainers must identify the particular joint strategy that yields the chosen outcome.
4. The **enforcement problem**: once the particular joint strategy is chosen the bargainers must identify an enforceable or self-enforcing agreement that implements the joint strategy.

B. One way to develop a theoretical analysis of bargaining is through **cooperative game theory**.

1. In a sense, cooperative game theory focuses on joint efficiency and payoff distribution, where non-cooperative game theory focuses on strategy coordination and enforcement.
 - a. What is sometimes called the "Nash Programme" is an attempt to link these two in an overall analysis of bargaining.
2. The analysis may, or may not, permit binding contracts.
 - a. It is, however, useful to think of the bargaining as being about a contract (or treaty).
 - b. However, it is important to remember that no party will enter into a contract unless they expect it to be beneficial.

3. Unlike the Non-Cooperative case, in cooperative theory we permit direct communication between players.

4. As a technical matter, we also permit players to jointly randomize.

a. This results in correlated strategies.

b. This permits the analyst to convexify the payoff space.

II. Nash Bargaining Theory

A. The Basic Strategy of Nash Bargaining Theory

1. We want to come up with a *general* characterization of the equilibrium contracts between players.

a. By general we mean that we want a rule (i.e. a function) that picks out solutions when presented with any bargaining problem of a well-defined class.

b. One way of thinking about this is that we want to describe the behavior of an ideal arbitrator.

2. Thus, rather than explicitly characterizing the choice process of the bargainers, as we generally do in non-cooperative game theory, we attempt to characterize the outcomes.

a. We do this by making a list of properties that we think a satisfactory outcome will have (the axioms of the theory).

b. Any rule that produces outcomes not consistent with these outcomes is henceforth excluded from our consideration.

(1) This has the obvious virtue of quickly reducing the number of schemes with which we need concern ourselves.

(2) Of course, we need to be careful not to impose a set of axioms which does not permit any outcomes.

c. John Nash (1950, 1953) proposed a small number of axioms with the striking result that, given certain technical assumptions, they pick out a unique outcome. The axioms of the Nash analysis served two different purposes:

(1) Some axioms embody the normative notions of what constitutes a

“good” solution; and

(2) Some axioms are used to extend the class of cases to which the solution refers to include certain types of “irrelevant” transformations.

B. The Axioms of Nash Bargaining Theory

1. Feasibility

a. We assume that there is a set of feasible outcomes.

(1) These will be determined by the particular, say, economic and political environment within which the bargaining takes place.

(2) We will represent this feasible set in utility space and denote it S .

(a) With two bargainers this means that the feasible set will be some subset of the u^1 - u^2 plane.

(b) We will assume that the relevant subset is compact (i.e. closed and bounded) and convex.

b. We assume that, within the feasible set, there is a point representing the payoffs received by each of the bargainers if no agreement is reached.

(1) This is sometimes called a “disagreement” or “reversion” point.

(2) We will denote the disagreement point d .

(3) We will also assume that there is at least one point that both players prefer to the disagreement point. That is, we assume that there is something to bargain about.

c. The feasible set and the reversion point completely define the game situation.

(1) This means that any information that we have about the players or the game situation must be embodied in either the feasible set and/or the reversion point.

d. Definition: A **bargaining problem** is a pair (S, d) , where $S \subseteq u^2$ is compact and convex; $d \in S$, and there exist $s \in S$ such that $s_i > d_i$ for all i .

(1) The set of all bargaining problems will be denoted B ; and

(2) A **bargaining solution** is a function $f: B \rightarrow \mathcal{U}^2$ that assigns to each bargaining problem (S, d) a unique element of S .

2. We should remind ourselves that, as in all of our analysis in this course, the **agents are rational**.

a. As with the consumer choice problem considered in the first lecture, preferences are taken to be a fundamental primitive.

(1) That is, each agent i has a preference ordering ξ_i over the feasible set S .

(2) Recall that ξ_i is a complete, transitive, continuous, binary relationship.

b. In our earlier analysis we were only concerned with the ordinal properties of utility functions. Thus, we assumed that those utility functions were unique up to any increasing transformation.

c. For our analysis of bargaining, we require a utility function with cardinal properties.

(1) Thus, our *utility functions are assumed to be unique up to an affine transformation*.

(2) That is, if u_i is a utility function that represents ξ_i , then v_i represents ξ_i iff $v_i = \alpha u_i + \beta$, with $\alpha > 0$.

d. Specifically, we will use *vonNeumann-Morgenstern* utility functions.

(1) This is a utility function defined on the set of lotteries over possible agreements, not just the agreements themselves.

(2) These utility functions embody each player's attitude toward risk and are unique up to a positive, affine transformation.

3. The bargaining problem as a game

a. **Players**: The bargainers $\{1, 2\}$

b. **Strategies**: Payoff demands $\{s_i: s_i \in \mathcal{U}^+\}$.

(1) Each player demands an individually feasible utility payoff.

c. **Outcomes:** If the demands are compatible (ie. jointly in the feasible set), each player receives his demand $s = (s_1, s_2)$; if the joint demands are not feasible, they get the disagreement outcome $d = \{d_1, d_2\}$.

d. **Structure**

- (1) One shot;
- (2) Simultaneous;
- (3) Complete information.

e. **Equilibrium:** The Nash Bargaining Solution

(1) It will turn out that the unique equilibrium to this game is the pair of demands $s = (s_1, s_2)$ that maximizes $(s_1 - d_1)(s_2 - d_2)$.

(2) We now turn to the proof of this claim.

4. **Normative Axioms**

a. *Pareto Efficiency* (PAR): The outcome should be (1) at least as good as the status quo; and (2) at least as good as any other feasible point.

(1) That is, contracts should be efficient in the sense of Pareto.

(2) This seems to be a fairly clear norm: players want to do as well as they can without hurting one another.

(3) Formally: If (S, d) is a bargaining problem with $s \in S$ and $t \in S$, and $t_i > s_i$ for all i , then $f(S, d) \dots s$.

b. *Symmetry* (SYM): If an abstract version of the bargaining game places the players in completely symmetric roles, then the outcomes should yield them equal utility payoffs in those roles. That is, we abstract from any difference in bargaining ability.

(1) This represents a norm of egalitarianism in the sense that the solution does not depend on any particular, unmodeled attributes of players (e.g. race, gender, name, etc.). Think about Rawls' original position in this context.

(2) This really follows from our earlier assertion that all the relevant

information about the bargaining situation is contained in (S, d) . But if players are interchangeable, then the bargaining solution must assign the same utility to each player if they are in the same role.

(3) Formally: If (S, d) is symmetric, then $f_1(S, d) = f_2(S, d)$.

5. Consistency Axioms

a. *Invariance to equivalent utility representations (INV)*: Player's preferences, not the specific utility functions used to represent them are basic.

(1) We noted in constructing utility functions that they are unique only up to an affine transformation. Thus, we don't want our solution to be affected by transformations of utility functions that are irrelevant to the strategic interaction.

(2) Formally: Suppose that (S', d') is obtained from (S, d) by the transformations $s'_i = \alpha_i s_i + \beta_i$, where $\alpha_i > 0$ for $i = 1, 2$. Then $f_i(S', d') = \alpha_i f_i(S, d) + \beta_i$.

b. *Independence of Irrelevant Alternatives (IIA)*: The idea here is that unchosen (i.e. irrelevant) options should not have an impact on the final solution.

(1) We can see this as representing some (unmodeled) bargaining process in which the negotiators gradually eliminate unacceptable outcomes until only one is left.

(2) Formally: If (S, d) and (T, d) are bargaining problems such that $S \subset T$ and $f(T, d) \in S$, then $f(S, d) = f(T, d)$.

(3) This turns out to be the most controversial of the axioms. Much current research in Nash bargaining revolves around the attempt to replace this axiom with something else.

(a) The problem is that we can all conceive of situations in which the presence of certain (non-chosen) alternatives affects the final outcome of a bargain.

(b) Concerns of this sort have led game theorists to attempt to formalize the bargaining process itself, thus moving cooperative game theory closer to the methods of non-cooperative game theory.

C. The Solution

1. **Theorem** (Nash 1950, 1953; *Etrica*): There is a unique bargaining solution, $f^N: B \rightarrow \mathbb{R}^2$ satisfying the axioms PAR, SYM, INV, IIA. It is given by

$$f^N(S, d) = \operatorname{argmax}_{(d_1, d_2) \leq (s_1, s_2) \in S} (s_1 - d_1)(s_2 - d_2).$$

a. The expression $(s_1 - d_1)(s_2 - d_2)$ is sometimes called the Nash product.

III. Alternatives to the Nash Bargaining Solution

A. Alternative 1: Nash with Individual Rationality

1. Individual rationality can be captured by the assumption that the utility pair picked out as the solution can be no worse than the disagreement point for either of the participants.

a. Strong Individual Rationality *SIR*: $f(S, d) \geq d, \forall d \in B$.

2. Note: Since f^d , which assigns every bargaining problem the disagreement point, is the only solution other than Nash's that satisfies SYM, INV, and IIA, and the condition that the solution give each player at least his disagreement utility, we can replace PAR with SIR in the statement of the Nash solution.

3. **Theorem** (Roth, 1977): The Nash solution is the unique function defined on B which satisfies SYM, SIR, INV, and IIA.

B. Alternative 2: The Asymmetric Nash Solution

1. We can replace SYM with SIR to get the Asymmetric Nash Solution.

2. **Theorem** (Harsanyi/Selten, 1972): For each vector $(t, 1-t)$, there is a unique bargaining solution, given by

$$g_t(S, d) = \max_{(d_1, d_2) \leq (s_1, s_2) \in S} (s_1 - d_1)^{1-t} (s_2 - d_2)^t,$$

subject to $u \in S$ and $u \geq d$.

3. **Example**: Bargaining between a National Government and a Multinational Firm

C. Alternative 2: The Kalai-Smorodinsky Solution

1. For every bargaining game (S, d) , define the ideal point, I_1 , by

$I_1 = \max \{s_1 : \text{for some } s_2 \geq 0, (s_1, s_2) \text{ is individually rational and feasible}\}$. Similarly for I_2 .

a. Ideal utility levels have the interpretation that they are the most that players can hope for assuming feasibility and individual rationality of their opponents.

2. The K-S Solution is a function f^{KS} that chooses, for every bargaining game (S, d) the unique Pareto optimal point

$$(s_1^*, s_2^*) : \frac{(s_1 - d_1)}{(I_1 - d_1)} = \frac{(s_2 - d_2)}{(I_2 - d_2)}.$$

a. Thus, the players choose the best outcomes subject to getting the same proportions of their ideal gains.

3. The most controversial of the Nash Axioms is IIA. K-S replace IIA with a *monotonicity* condition, while retaining the other axioms.

a. *MON*: If $S \subseteq T$ and if, for $i = 1, 2$, the maximum utilities that player i can obtain in $\{s \in S : s \geq d\}$ and $\{s \in T : s \geq d\}$ are the same, then each player receives at least as much utility in the solution of (T, d) as in the solution of (S, d) .

b. This essentially says that if the feasible set is changed in favor of one of the players, then that player should not lose as a result of the change.

4. **Theorem** (Kalai-Smorodinsky, 1975): The K-S solution is the only solution satisfying SYM, PAR, INV, and MON.