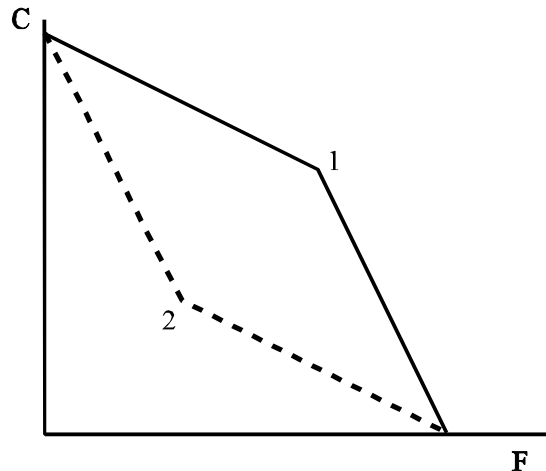


## Homework #4: Answers

Text questions, Chapter 5, problems 1-5:

1. Draw the array of world outputs that free trade allows by making use of each country's transformation schedule.

a. Draw it.



This diagram is constructed by recognizing that maximum world output of C is just the vertical sum of maximum national outputs. From that point, if there is any demand for F, it will be provided by the country with a comparative advantage in F production. In the text, this is the Foreign country. Thus, the slope of the world PPF will be identical to that of the Foreign country's PPF. Foreign will continue to provide F (as well as C, Home is specialized to C production), until it is specialized in the production of F, at point 1. From that point, additional demand for F must come from the Home country, which will be producing both goods until it too is specialized in production of F.

b. What is the menu of the worst combinations of outputs? Show that it is made up of two linear segments.

The dashed line in the above diagram is the menu of nationally efficient but globally inefficient points. [i.e. the truly worst point would be the origin—zero output from either country.] This is generated by perverse specialization. That is, from the point of globally maximum output of C, if demand for F is first served by the Home country, until it is specialized, at point 2, and then by the Foreign country, we construct the dashed frontier.

2. In the discussion of Section 5.6 suppose that both Home and Foreign countries produce commodity 3.

Recall that the situation in section 5.6 involves a large country producing 3 goods (1,2,3),

and a second country producing good 3, and possibly other goods.

- a. What can be said about the distribution of income between countries if Home demand switches a small amount from commodity 2 to some commodity produced abroad?

Answering these questions involves using the zero-profit conditions—i.e.  $P_j \leq a_{Lj}w$  and  $P_j^* \leq a_{Lj}^*w^*$ —and recognizing that if good 3 is produced in common  $P_3 = P_3^*$ . Using these two facts, and fixed  $a_{Lj}$ , we get a key implication:  $P_3 = P_3^* \Rightarrow a_{Lj}w = a_{Lj}^*w^*$ , so if price of good 3 is unchanged between equilibria, wages must be unchanged. Thus, there are two cases to consider. First, the increased demand for Foreign labor (in the form of increased demand for Foreign goods) raises  $w^*/w$ . But this must result in  $a_{L3}w < a_{L3}^*w^*$ , and a shift of all good 3 production to the Home country. Alternatively, the Home country is large and Foreign is specialized to production of good 3, before and after the changed demand. Because relative prices are determined by production conditions in the large (i.e. Home) country, relative prices are unchanged, both countries continue to produce good 3, so  $w^*/w$  is unchanged.

- b. If Home technology for producing commodity 1 improves?

Fix the price of good 3 at \$1. Since there is no change in technology in the production of good 3, there will be no change in wages in either country. The price of good 1 will then fall by an amount equal to the amount of the productivity increase in production of good 1. This will lead to gains for both countries, with the gains being proportional to the amount of good 1 *consumed*.

3. Suppose costs of production depend only on labor costs, and that to produce a unit of each commodity in each country takes the number of labor-hours shown:

	$a_{LA}$	$a_{LB}$	$a_{LC}$
Home	10	10	10
Foreign	3	5	7

- a. In which commodity does the Home country possess the greatest comparative advantage?

For making comparisons with  $n > 2$  goods, it is useful to recall that unit cost for good  $j$  is  $c_j = a_{Lj}w$ . Furthermore, with trade the law of one price implies that there can only be one price for good  $j$  and  $p_j = c_j$  in all countries with positive output. Thus, if  $a_{Lj}w < a_{Lj}^*w^*$ , then only the Home country will produce good  $j$ . Rearranging, we have a condition which states that if:

$$\frac{a_{Lj}}{a_{Lj}^*} < \frac{w^*}{w} \Rightarrow \text{Home produces } j.$$

Similarly, if  $a_{Lj}w > a_{Lj}^*w^*$ , then only the Foreign country will produce good  $j$ , and we have:

$$\frac{a_{Lj}}{a_{Lj}^*} > \frac{w^*}{w} \Rightarrow \text{Foreign produces } j.$$

Taking these together means that we can create a chain, ordering industries by degree of Home advantage. Using the data in the above table, we have:

$$\frac{a_{LC}}{a_{LC}^*} = \frac{10}{7} < \frac{a_{LB}}{a_{LB}^*} = \frac{10}{5} < \frac{a_{LA}}{a_{LA}^*} = \frac{10}{3}.$$

which implies that the Home country has the greatest comparative advantage in the production of commodity  $C$ . Note that these ratios are absolute advantage comparisons and are not informative about production and trade without wage data. But we can use this chain to identify the extreme members.

- b. If the Foreign wage rate is \$1 per labor hour and a free-trade equilibrium is reached, what is the most that the Home wage rate can be? Why?

Recall that which countries produce which goods is determined by the relative wage in the two countries  $w^*/w$ . With a Foreign wage of \$1, the Home country must be able to produce at least one good, so the  $w^*/w$  cannot be less than 10/7. With a Foreign wage of \$1, the Home wage rate can be at most 7/10.

- c. If the Foreign wage rate is \$1 per labor-hour, what would a possible Home wage rate be so that the Home country can produce only one commodity? Which commodity would that be?

This question asks to find a Home wage, given a Foreign wage of \$1, such that Home is just priced out of producing good B. This will occur if  $w^*/w > 2$ . Thus, the Home country can produce only one commodity,  $C$ , for any wage greater than 1/2.

4. In a Ricardian world, with labor the only factor of production being paid, the following table gives the constant labor costs per unit of producing different commodities for countries  $\alpha$  and  $\beta$ .

	Wheat $a_{LW}$	Cars $a_{LC}$	Tankers $a_{LT}$	Reactors $a_{LR}$	Tractors $a_{LX}$
$\alpha$	10	10	10	10	10
$\beta$	5	8	10	12	14

a. In which commodities does country  $\alpha$  have an absolute advantage? Why?

Country  $\alpha$  has an absolute advantage in the production of reactors and tractors. Recall that absolute advantage involves a direct comparison of labor productivities. Since  $a_{LR} < a_{LR}^*$  and  $a_{LX} < a_{LX}^*$  we have our answer. Note that we could compare labor productivities directly by looking at MPPLs:  $\alpha_{LR} > \alpha_{LR}^*$  and  $\alpha_{LX} > \alpha_{LX}^*$ .

b. In which commodities does country  $\beta$  have a comparative advantage? Why?

As in the previous question, we need to construct the chain:

$$\frac{a_{LX}^{\alpha}}{a_{LX}^{\beta}} < \frac{a_{LR}^{\alpha}}{a_{LR}^{\beta}} < \frac{a_{LT}^{\alpha}}{a_{LT}^{\beta}} < \frac{a_{LC}^{\alpha}}{a_{LC}^{\beta}} < \frac{a_{LW}^{\alpha}}{a_{LW}^{\beta}}$$

$$\frac{10}{14} < \frac{10}{12} < \frac{10}{10} < \frac{10}{8} < \frac{10}{5}.$$

But, without information on relative wages, we cannot say where the chain will be broken.  $\beta$  will certainly have a comparative advantage in wheat, but beyond that we cannot say.

c. Which country would export tankers? Explain.

Country  $\alpha$  will export tractors, if it exports anything. Tankers, in the middle of the chain cannot be predicted without information on  $w^{\beta}/w^{\alpha}$ .

5. Consider the world to consist of two countries (Home and Foreign) made up of individuals with identical and homothetic taste patterns. Portray these by a set of smoothly bowed-in indifference curves. Suppose production in the Home country requires two labor hours per unit of food and only one labor-hour per unit of clothing, whereas the Foreign country's figures for food and clothing are just the opposite. The Foreign country's labor force consists of 1 million labor hours.

a. If the Home country is small relative to the Foreign country, one of the countries will produce both goods. Which country? What will be food's relative price?

It is useful to start by explicitly showing the opportunity cost ratios:

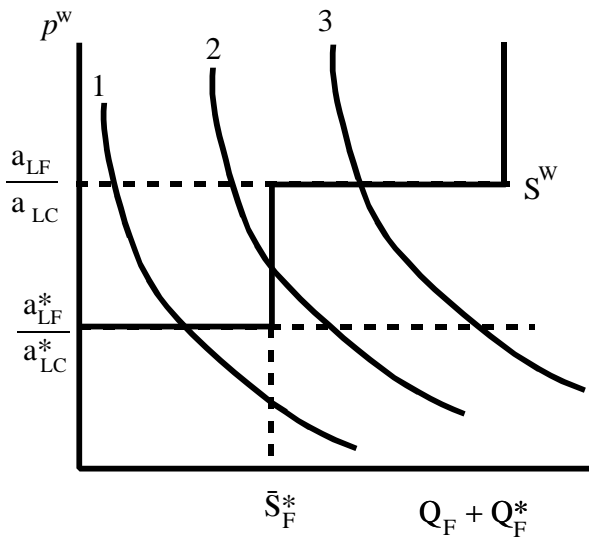
$$\frac{a_{LF}^{Home}}{a_{LC}^{Home}} = \frac{2}{1} > \frac{1}{2} = \frac{a_{LF}^{For}}{a_{LC}^{For}}.$$

The small country always specializes. Since the small country is a price taker, the world price is fixed by the large country's autarky price ratio. Thus, if Foreign is large, the world relative price of Food is  $\frac{1}{2}$  and the Home country will specialize in clothing. The Foreign country will produce both goods.

- b. If the Home country is large relative to the Foreign country, one of the countries will produce both goods. Which country? What will be food's relative price?

The situation here is just reversed. The world price is now set by the, now large, Home country's autarky relative price of food, which is 2. The, now small, Foreign country will specialize in its comparative advantage industry, in this case food. And the Home country will produce both goods.

- c. Construct world supply and demand schedules for food, with  $p = P_F/P_C$  on the vertical axis and world food output on the horizontal.



The supply curve has two horizontal components and one vertical. For  $D^W \leq \bar{S}_F$ , e.g. 1, that demand will be satisfied by the country with a comparative advantage in food, i.e. the Foreign country. At the Foreign specialization point,  $\bar{S}_F$ , there are a range of prices consistent with each country being specialized in its comparative advantage good—e.g. world demand curve 2. If demand is sufficiently great that output of food from the Home country is also required, i.e.  $D^W \geq \bar{S}_F$  (e.g. at demand curve 3), the world price will be set by technical conditions in the Home country. This determines the other horizontal component.

Workbook questions, 1, 2, 4, and 8

1. *Absolute versus Comparative Advantage*: Suppose that in the US four man-hours are required to produce each unit of clothing and each unit of food. In Canada, one man-hour is required for each unit of clothing and two man-hours are required for a unit of food.

- a. Which country has an absolute advantage in each good?

To start with, it is useful to collect these assumptions into a simple table:

	$a_{LC}$	$a_{LF}$
US	4	4
Canada	1	2

Note that absolute advantage comparisons involve bilateral comparisons of productivity by industry. Thus, Canada has an absolute advantage in the production of both goods, i.e.:  $1 < 4$ , and  $2 < 4$ .

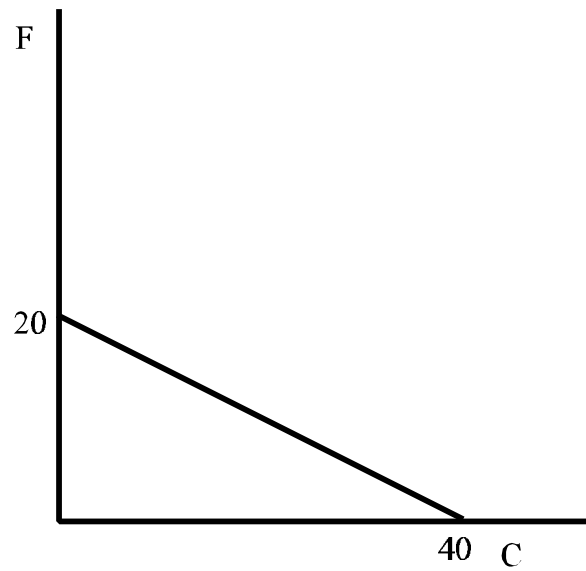
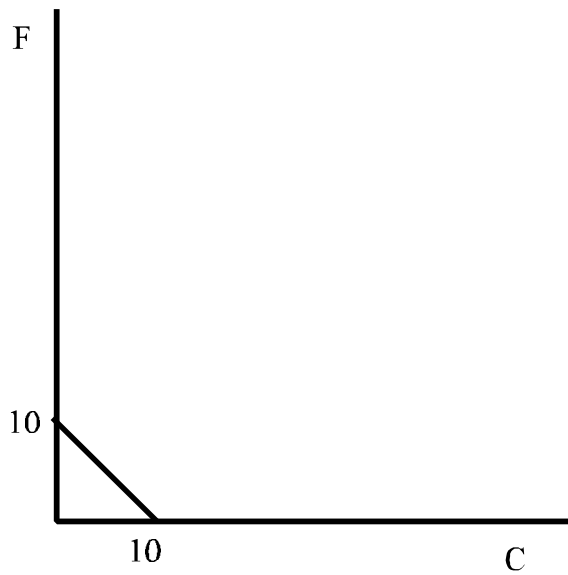
- b. Which country has a comparative advantage in each good?

Recall that comparative advantage comparisons involve comparing relative productivity. Thus, we need to compare:

$$\frac{a_{LC}^{US}}{a_{LF}^{US}} = \frac{4}{4} = 1 > \frac{1}{2} = \frac{a_{LC}^{Can}}{a_{LF}^{Can}}.$$

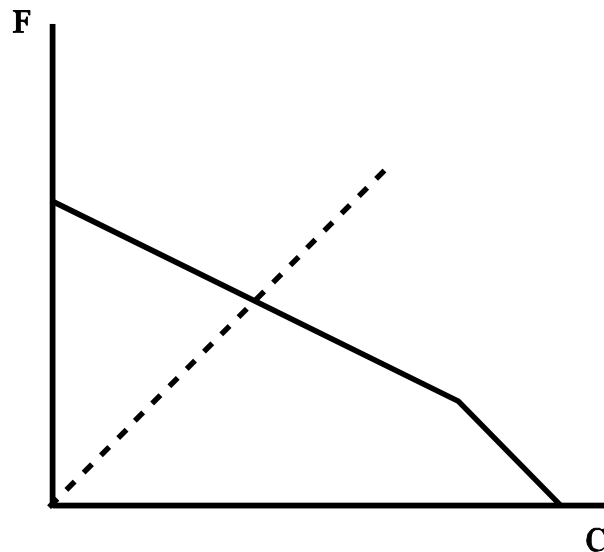
This comparison shows that Canada has a lower opportunity cost/autarky price of clothing. Thus, Canada has a comparative advantage in clothing.

- c. Assuming that each country has 40 man-hours of labor available for production, draw the production possibilities frontiers for each country. (Put food on the vertical axis). What do the slopes of these frontiers indicate?



As we derived in class, the slopes of these curves show the opportunity costs of clothing in terms of food. That is, the number of units of food that must be given up to secure one more unit of clothing.

- d. Draw the world production possibilities frontier. What does its slope indicate?



As drawn, the world production possibilities frontier shows output being produced by the most efficient country. Just as in the individual countries, the slope shows the opportunity cost of clothing in terms of food, in this case to the world economy as a whole. Since equilibrium occurs on the part of the world PPF whose slope is determined by Canadian production conditions, this means that the world terms-of-trade will be equal to the

autarky MRT in Canada. This implies, by the way, that all gains from trade will accrue to the US.

- e. If consumers in both countries have Leontief preferences, consuming clothing and food in fixed proportion of one-to-one, what is the trade pattern?

If we draw in the income-consumption line, which is fixed for Leontief preferences, we see that Canada will produce both goods, while the US will be specialized to food. Thus, the US will export food and import clothing, while Canada does the reverse.

- f. If the labor force in the US increases by a factor of 20, will anything happen to the trade pattern?

This increase in the endowment of labor will result in a new world equilibrium, in which the US is producing both goods and Canada is specialized to clothing, with the world price determined by the opportunity cost ratio in the US. However, the trade *pattern* remains the same: US will export food and import clothing, while Canada does the reverse. This is determined by comparative advantage, a fact determined by technology, which was unchanged by the increase in US population.

2. *Commodity prices and factoral terms of trade*: Suppose the labor coefficients in the food and clothing industries are as follows:

$$\begin{array}{ll} a_{LC} = 4 & a^*_{LC} = 10 \\ a_{LF} = 8 & a^*_{LF} = 5 \end{array}$$

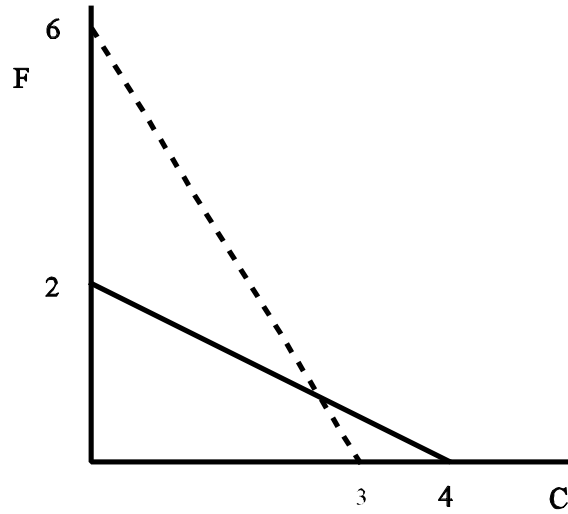
Suppose also that the Home country has 16 units of labor and the Foreign country has 30.

- a. Draw the production possibilities frontier for each country and calculate each country's autarky prices.

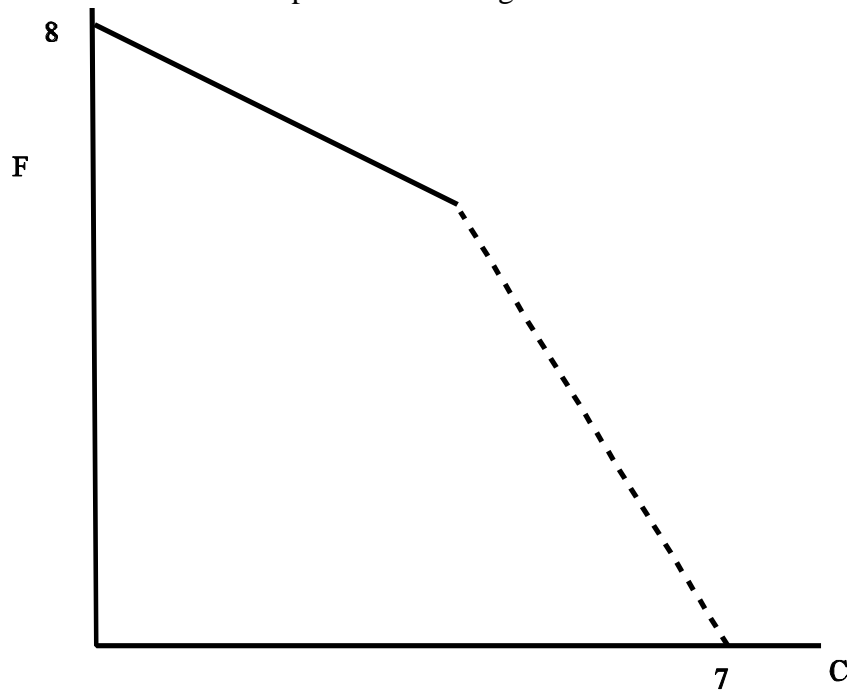
Since autarky relative prices will be equal to the opportunity cost ratio (i.e. the MRT) we can calculate the prices as:

$$\frac{P_C}{P_F} = \frac{a_{LC}}{a_{LF}} = \frac{4}{8} = \frac{1}{2} \quad \text{and} \quad \frac{P_C^*}{P_F^*} = \frac{a^*_{LC}}{a^*_{LF}} = \frac{10}{5} = \frac{2}{1}.$$

Thus, autarky price of clothing in the Home country is ½ unit of food, while the autarky price of clothing in the Foreign country is 2 units of food. The PPFs for the two countries are shown in the following graph.



- b. Draw the world production possibilities frontier. What must the world relative price of food in terms of clothing be in order for both countries to specialize in food? In clothing? Over what range of prices will countries specialize in the good in which each has a comparative advantage?



$\frac{P_F^W}{P_C^W} > 2$  will result in both countries specializing in food, and  $\frac{P_F^W}{P_C^W} < \frac{1}{2}$  for both countries to specialize in clothing. If  $\frac{1}{2} < \frac{P_F^W}{P_C^W} < 2$  each country will specialize to its

comparative advantage good.

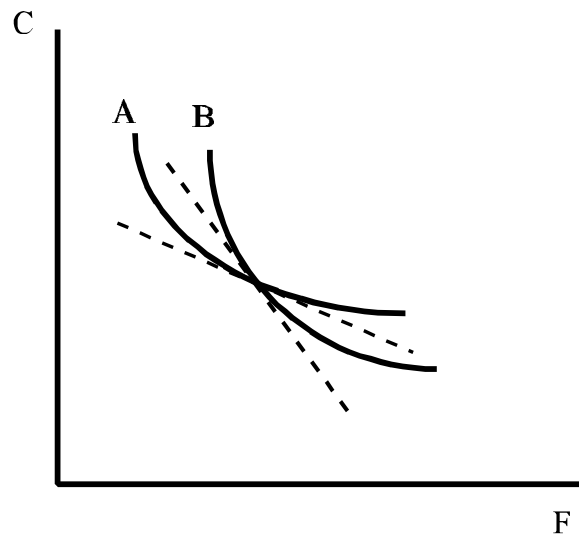
- c. Suppose that the world price ratio is equal to 1. Calculate the relative wages,  $w/w^*$ .

If prices are equal, both countries are specialized to their comparative advantage goods and  $P_C = a_{LC}w = a_{LF}^*w^* = P_F^*$ , so  $4w = 5w^*$  and  $w/w^* = 5/4$ .

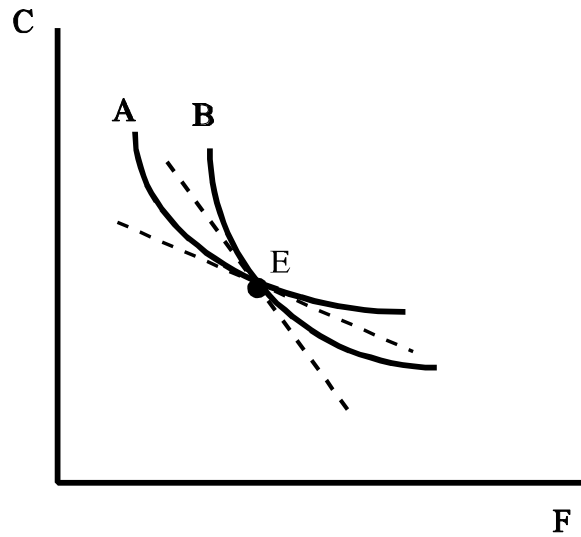
- d. Now suppose that there is an increase in world demand for clothing so that the relative price of clothing rises to  $3/2$ . What happens to the relative wage between the Home and Foreign country?

Patterns of production and specialization are unaltered, i.e. both countries are specialized in their comparative advantage good. However, now commodity prices are not equal, but are related as  $3/2$ . Thus the relative wage,  $w/w^* = 15/8$ .

4. *Comparing models*: Define an increase in demand for clothing as a shift in the indifference curves toward the clothing axis. This is illustrated below as a shift from curve A to curve B.

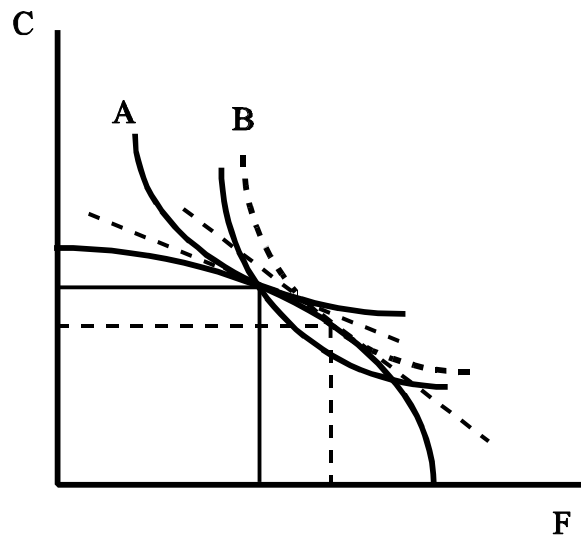


- a. Consider first an endowment economy with an allocation for food and clothing shown by point E and the indifference curve A. Given our definition of a “demand shock”, suppose there is an increase in demand for clothing. What happens to the relative (autarky price of clothing)? What happens to the supply of clothing.



From the above diagram it is easy to see that at tastes given by B the relative price of food (determined by the MRS at E) is greater than when the tastes are given by A. Because this is an endowment economy, there is no change in supply.

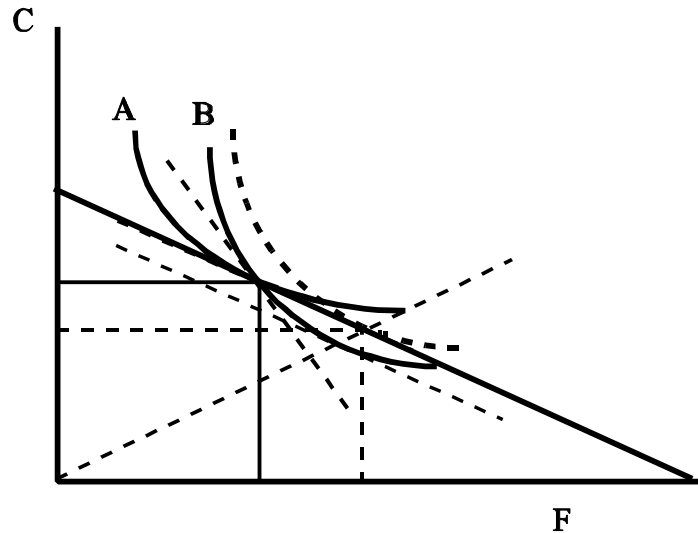
- b. Now suppose the economy has a bowed out production possibilities frontier. What happens to the relative price of clothing when demand for clothing increases? What happens to the supply of clothing?



Now the final price is determined by the tangency between an indifference curve and the PPF at the point where  $MRS = MRT$ . At this new equilibrium point the relative price of food has increased, but so has the output of food. The output of clothing has, of course,

fallen.

- c. Finally consider a Ricardian model of production. Now what happens to the relative price of clothing when the demand for clothing increases? What happens to the supply of clothing?



In this case, because we are at an interior equilibrium both before and after the change in tastes (i.e. both goods are produced in both equilibria), the relative price is unchanged. Recall that in the Ricardian model, relative price at interior equilibria is completely determined by technological conditions. Adjustment occurs only on the output margin, with output of food rising and output of food falling.

- d. Since we know that autarky prices determine the pattern of trade (i.e. via the law of comparative advantage), in what sense are the endowment model and the Ricardian model “extreme cases”? (Hint: Think about the role of supply and demand in determining autarky prices.)

In the endowment model autarky prices are determined completely by the MRS, while in the Ricardian model they are determined completely by the MRT. That is, demand conditions determine autarky relative price in the first case and supply conditions determine autarky relative price in the latter case. In the standard neoclassical case, the interaction of these two forces determine the autarky relative price.

8. *International wage comparisons:* The following table of wage rates in different countries was published in the *Wall Street Journal*.

Hourly Pay Levels  
(For production workers in US Dollars)

	1985	1986	1987	1988
United States	\$12.96	\$13.19	\$13.44	\$13.62
Japan	\$6.47	\$9.47	\$11.44	\$13.80
West Germany	\$9.56	\$13.35	\$16.87	\$20.19
Italy	\$7.40	\$10.01	\$12.33	\$14.77
France	\$7.52	\$10.27	\$12.42	\$14.03
Britain	\$6.19	\$7.50	\$8.96	\$11.06

In view of what you have learned from the Ricardian model of production, how would you respond to the following comments?

- a. The high cost of hiring production workers in the US has undermined its competitive position in world markets.

No. US workers are paid more because they are more productive.

- b. The high wages in the US compared to those found abroad are due to labor unions forcing their levels above what would be implied by labor's productivity in a competitive Ricardian world.

Possible, but not likely. If wages were held above competitive levels, US goods would be unable to compete internationally, causing the eventual shutdown of industries supporting such wages. Also, note that this might explain the relatively high wages in 1985, but if this explanation is correct, the data show a weakening of unions in the US relative to the other countries, where much stronger wage growth is seen.

- c. The increase in Japan's wages implies that Japan is losing its competitive edge in world markets.

No. It implies that the productivity of Japanese workers is increasing.

- d. The low wages in Britain are an indication that British workers are being underpaid.

No. Their productivity is lower.

- e. Allowing importation of goods from workers with lower wages is unfair to American workers.

No. The lower paid worker is also the less productive one.