Homework #5: Answers

Text questions, Chapter 6, problems 1-4. Note that in all of these questions, the convention in the text, whereby production of food uses land and labor, and clothing uses capital and labor, is adopted. The two main diagrams are given at the end.

1. The discussion of the Ricardian model in Chapter 5 introduced the concept of an input-output coefficient, $a_{ij}$. The reciprocal of this ($1/a_{ij} = a_{ij}$) referred to the average product of labor.

   a. In a diagram that shows the marginal product of labor, draw in a curve showing the average product of labor.

   We start by showing the relationship between marginal and average products of labor using the graph of the production function. Marginal product is shown as the slope of the production function graph for any allocation of labor, and the average product is shown by rays from the origin through points on the production function for those same points. It should be clear that the average product line is always steeper than the marginal product lines. Thus, in the diagram illustrating the marginal product, the average product curve will lie everywhere above the marginal product curve.

   b. How can land rents as well as total wages be shown in such a diagram?

   One way to do this is to recognize that, in equilibrium, $w = VMP_L$, and that the total wage bill is equal to $wL$, which is the rectangular area with the backslash shading. Total product is $L \times VAP_L$, which is the total rectangular area. If we net out the payment to labor, the remainder is the payment to capital. However, a more rigorous approach also begins by recognizing $w = VMP_L$, and that the total wage bill is equal to $wL$. However, now we note, from the fundamental theorem of calculus, that the area under a marginal curve is equal to the value of the total function. Thus, integrating under the $VMP_L$ curve between 0 and $L'$ gives the value of total output. Netting out the payment to labor again yields the total payment to capital.
2. With reference to figure 6.2 (the labor market diagram) it was suggested that a 10 percent increase in the price of food would shift the $VMP_L^F$ curve upward by 10 percent, while a 10 percent increase in the supply of land would (at constant food and clothing prices) shift the $VMP_L^F$ curve rightward by 10 percent.

![Diagram of labor market with VMPL curves and wage rates]

**a.** Do these have equivalent effects on the wage rate?

Both changes will cause the $VMPL_L^F$ curve to shift upward. In general, a 10% increase in the endowment of land will not shift the $VMPL_L^F$ curve by 10%. If, by some chance, it did, then the effect on the wage (and the return to capital) would be the same. The effect on returns to land would be different: landowners gain from the price increase, but lose from the endowment increase. It is useful to note that the wage must rise by less than 10 percent. If the allocation of labor after either of the changes remained unchanged at $L'$, the returns to both labor and land would rise by 10%. That is, the increase from $w'$ to $w''$ is an increase of 10%. However, the increased wage in the food sector attracts labor into food, from clothing, until the wage is equalized between sectors. Given the diminishing marginal product in both sectors, the increased labor in food depresses $MPPL_F$ and thus $w$, and increases $MPPL_C$. Thus, the wage rises by less than 10%, to $w''$.

**b.** Which kind of change would workers prefer?

In the case of increased productivity, with prices constant, workers experience a gain in real income (i.e. $w$ increases relative to both commodity prices). In the case of a price increase, workers experience an increase relative to (unchanged) clothing prices, but a decrease in purchasing power relative to the price of food. Thus, workers would prefer the technology change to the price change.
c. Which would capitalists prefer?

Capitalists are indifferent. In either case, the nominal wage rises to $w''$, which must reduce the return to capital (i.e. the specific factor in clothing production).

3. Explain why Australian capitalists and landlords probably favor the same policy toward immigration.

a. Explain it.

If labor is the mobile factor, immigration would appear as an increase in the labor endowment. Specifically, in the labor market diagram, this would be an increase in the length of the diagram’s base by the amount of the immigration. It is easy to see in the diagram that this will result in a lower wage and a higher rental for both fixed factors (i.e. for capital and land).

b. Given the traditional export position of Australian wool in world markets, how might owners of sheep stations be expected to react to an increase in domestic prices of manufactures brought about by a tariff?

An increase in the domestic price of manufactures (clothing, in the above picture) would shift up the $VMPL_C$ curve by the proportional increase in price. This will raise the wage and, at unchanged wool prices, this will depress the return to land (or sheep, whatever is the specific factor in wool production). Thus, owners of sheep stations would be expected to oppose the tariff.
c. Through what mechanism might land rents be disturbed?

By direct comparison, an export subsidy or a production subsidy would raise the price to wool producers, thus raising the return to land and depressing the return to capital.

4. Contrast the effect on land rents of an increase in a nation’s supply of land coupled simultaneously with a reduction in its supply of capital if:

a. The country cannot engage in world trade.

In autarky, relative prices are determined by the interaction of supply and demand conditions. The effect of these endowment changes is to rotate the production function for food (i.e. the sector using land) outward, and rotate the production function for cloth (i.e. the sector using capital) inward. This will cause the PPF to shift out along the food axis and in along the clothing axis. If C is on the horizontal axis, this will cause the relative price of clothing to rise. Shown as the tangency between the indifference curve and the PPF. In the labor market diagram there are, thus, two kinds of effects: the VMPL\textsubscript{F} curve is shifted up by the increase in endowment, while the VMPL\textsubscript{C} curve is shifted down; however, the increase in the relative price of clothing shifts the VMPL\textsubscript{C} up. Thus, the total effect on land owners is ambiguous.

b. The country does trade freely with a much larger world market.

Recall that the small country assumption locks in the prices facing the small country. Thus, the two endowment changes will lead to an unambiguous increase in total returns to land. However, the increase in the endowment of land makes return per unit land ambiguous.

c. Answer the same two part question if the nation’s supply of land remains constant while its supply of capital rises.

In this case, only the production function for C will rotate outward, causing the PPF to shift outward along the C axis, with an unchanged F intercept. In the closed economy
case, this will reduce the relative price of \( C \). The first effect shifts the \( VMP_L^C \) curve up, while the second shifts it down. Thus, the effect on land owners is ambiguous. In the small open economy case, the \( VMP_L^C \) curve will shift up, causing an increase in the wage and a reduction in the return to land owners.

Workbook problems, 2-6

2. Determination of Factor Prices in a Small Open Economy: In each of the cases below, use the labor market diagram to draw in the shift(s) and determine the change in the wage rate relative to the price of clothing and the price of food.

a. A 10% increase in the supply of capital.

This causes a shift upward in the \( VMP_L^C \) curve, which will raise the wage relative to the price of food and clothing. Recall that prices are unchanged.

b. A 10% increase in the supply of labor.

This causes an increase in the base of the diagram, which will result in a decrease in the wage relative to both prices.

c. A 10% increase in the (absolute) price of food, holding the price of clothing constant.

This cause an upward shift in the \( VMP_L^F \) curve, and an increase in the wage relative to the price of clothing, but a decrease relative to the price of food.

3. Changes in the Distribution of Income: The legislature of a small country engaged in trade is contemplating a tariff on imports of food that will raise the domestic price of food by 10%. Food production requires labor and land, clothing production requires labor and capital.

a. Rank the changes in the returns to land and capital (\( r_T \) and \( r_K \)) and labor (\( w \)) relative to the change in the price of food.

\[
\hat{r}_L > \hat{P}_F = 10\% > \hat{w} > \hat{P}_C = 0 > \hat{r}_T.
\]

This can be explained using the labor market diagram, shifting the \( VMP_L^C \) curve up by 10%, and tracing through the effects of factor returns.

b. Do landlords stand to gain unambiguously from the proposed tariff? Why/why not?

Landlords gain unambiguously because their factor return increases relative to both consumption good prices.
c. Must capitalists see their returns fall? Why/why not?

Capitalists lose unambiguously because their factor return falls relative to both consumption good prices.

d. Suppose that workers, landlords and capitalists all have the same taste patterns, and that the country imports food. Explain how laborers would react to the proposal.

Laborers gain relative to the price of clothing and lose relative to the price of food. Thus, their position on the tariff will depend on the relative significance of food and clothing in their consumption bundle.

e. Who would support a political action committee (PAC) to promote the tariff?

Landlords would support the PAC, labor may or may not depending on the effect of the tariff on their welfare.

4. The Four-Quadrant Diagram: The diagram below is a reproduction of Figure 6.1 in the textbook. In drawing the diagram, it was assumed that capital is specific to the clothing sector, land is the factor specific to the food sector, and labor is mobile between sectors.

a. Suppose this economy experiences an increase in the supply of capital.
i. Draw in the shift(s) in the total product of labor curve(s) and the resulting shift in the production possibilities frontier.

The graph of the production function for clothing will rotate outward. This will cause the PPF to shift out along the clothing axis, but the food axis intercept will be unchanged.

ii. Assuming homothetic tastes, what happens to the relative price of clothing?

The new equilibrium, defined by the MRT = MRS = ρ condition, must involve a lower relative price of clothing.

iii. What happens to the allocation of labor between the sectors?

Labor is reallocated from food to clothing. Note that, for any relative output (i.e. a ray from the origin through a production point), the PPF after the increase in capital will be flatter: i.e. MRT’ > MRT”. But, from homotheticity, this means that along the ray through the original equilibrium (where MRT’ = MRS’), where that ray intersects the new PPF, MRS’ > MRT”. This means that the utility of C relative to F is greater than the opportunity cost of C. By the arbitrage argument, this will lead to increased output of C. But increased output of C means a reallocation of labor from F production to C production.

b. Now suppose that the economy experiences an increase in the supply of labor instead of capital.

i. Draw in the shift in the labor constraint and the resulting shift in the PPF.

In this case, the labor constraint shifts to the southwest. Since labor is used by both sectors, the PPF will shift out along both axes, and the new PPF will lie everywhere outside the old one. However, with land and capital fixed, and specific to their respective sectors, this shift need not be symmetric.

ii. Assuming homothetic tastes, what happens to the relative price of clothing?

Even with homothetic preferences, we cannot predict the final price, because we cannot say what will happen to the new PPF. If the shift were symmetric, then there would be no change in relative price. If the shift is greater along the C axis, as in the previous question, the relative price of C will fall; and if the shift is greater along the F axis, the relative price of C will rise.

iii. What happens to the allocation of labor between the sectors?

As in (b.ii), we do not have sufficient information to answer this question. If the shift were symmetric, homothetic preferences would ensure that the share of labor allocated to
each sector would be unchanged—i.e. the new labor would be allocated between the two sectors in the same proportions as characterized the original allocation. However, depending on the orientation of the new PPF, the new allocation could involve proportionally more or less labor in C.

5. The Netherlands—1992, and a little Economic History: The Netherlands exports agricultural products and imports manufactured goods from the European Union. For the purposes of this problem, assume that the Netherlands is small relative to the size of the European market. Suppose also, that the production of agricultural products requires labor and sector-specific land, while manufacturing uses labor and sector-specific capital as inputs.

a. Suppose that before 1992, the Netherlands had border controls with effects roughly equivalent to those of a tariff on imports. What would have been the attitude of Dutch economic groups towards the elimination in 1992 of all such controls within the European Union?

Use the labor market diagram and note that a reduction in protection on the importable (manufactures) shifts down the \( VMPL_M \) curve. We can trace through the income distribution effects as:

\[
\hat{r}_T > \hat{P}_A = 0 > \hat{w} > \hat{P}_M > \hat{r}_K.
\]

i. Workers

Workers gain in terms of the price of manufactures but lose in terms of agricultural goods.

ii. Landowners

Landowners gain unambiguously.

iii. Capitalists

Capitalists lose unambiguously.

b. The European Commission announced that migration of labor between member countries should be unrestricted by 1992. Given that Dutch workers earned some of the highest wages within the European Union, what would have been the attitude of economic groups towards an integrated European labor market.

i. Workers

The increase in labor will cause the wage to fall relative to the unchanged (by the small country assumption) prices of both consumption goods. Thus, labor should oppose more integrated labor markets.
ii. Landowners and Capitalists

Both landowners and capitalists gain in real terms, so they can be expected to support more integrated labor markets.

c. In the past, the Dutch have skilfully conquered land from the sea. As an economic historian who believes that the specific factors model is the appropriate model for analyzing the Dutch economy, do you think that the construction of each new dike was applauded by every citizen of the Netherlands?

The key to answering this question lies in recognizing that dikes are part of the land reclamation process. Thus, to answer this question, use the labor market diagram to evaluate the effect of an increase in land (specific to agricultural production). If commodity prices are fixed, labor will gain unambiguously (i.e. the wage rises, and prices are unchanged). Capitalists lose unambiguously (i.e. the increase in the wage reduces the total rent and, with a fixed quantity of capital, this must lower the return per unit capital). Landowners will also lose: price is unchanged, so the increase in wage must be balanced by a reduction in the rental rate per unit of land. Thus, labor should support dike building, while capitalists and landowners oppose it. Do you think this is what happened?

6. Sector-Specific Labor–Reinterpreting the Specific-Factors Model: Consider an economy with three types of workers: one group of workers with specialized skill in producing manufactured goods, one group with specialized skills in making textiles, and another group which can work equally well in both industries but with no specialized skills in either industry. Define their wages as $w_M$, $w_T$, and $w_B$. In each part, use graphs or equations to support your answer.

a. Suppose that there is an immigration of workers with specialized skills in producing manufactures. What happens to the production of manufactures? of textiles?

Using the labor market diagram, an increase in $S_M$ shows up as an upward shift in the $VMP_{LM}$ curve. The new equilibrium involves a shift in the allocation of labor from textiles to manufactures. This means that the output of manufactures rises, while the output of textiles falls.

b. At unchanged prices of manufactures and textiles, what happens to the wages of the three groups as a result of this immigration?

With fixed commodity prices, unskilled labor gains unambiguously, and both skilled labors lose. Skilled textile labor loses because the increase in the wage bids down the aggregate rent, while skilled manufacturing labor loses (fixed price with an increased unskilled wage must imply a reduction $w_M$).

c. Suppose instead that there is an increase in immigration of unspecialized workers. What happens to the production of manufactures and textiles?
This results in an increase in the equilibrium output of both sectors.

d. What type of immigration policy would specialized workers prefer? If this model were an accurate description of the U.S. economy, what types of American workers would be most concerned about the immigration of unskilled workers into the United States?

Specialized workers would prefer an immigration policy that admits only unspecialized workers. Other unskilled workers would oppose this policy.
The 4-Quadrant Diagram
The Labor Market Diagram