Homework #6: Answers

Text questions, Chapter 7, problems 1-2.

1. Suppose there is only one technique that can be used in clothing production. To produce one unit of clothing requires four labor-hours and one unit of capital; in food production each unit requires a single labor-hour and one unit of capital. At an initial equilibrium suppose the wage rate and the capital rental are each valued at $2.

a. If both goods are produced, what must be their prices?

Note that these are Leontief (i.e. fixed coefficient) production functions. The zero profit conditions for these industries are:

$$\begin{aligned} P_F &= a_{LF}w + a_{KF}r = (1\times 2) + (1\times 2) = 4 \\ P_C &= a_{LC}w + a_{KC}r = (4\times 2) + (1\times 2) = 10 \end{aligned} \qquad \textbf{(1)}$$

The second equality follows from substitution. The, if both goods are produced, $P_F = 4$ and $P_C = 10$.

b. Now keep the price of food constant and raise the price of clothing to $15. Trace through the effects on the distribution of income. Rank the relative changes in the wage rate, the price of clothing, the price of food (unchanged by assumption), and the rent on capital. Relate your results to the Stolper-Samuelson theorem.

This problem assume a 50% increase in P_C, i.e. $\frac{P_C'' - P_C'}{P_C'} = \frac{15-10}{10} = .5$. It is assumed that clothing is the L-intensive good, i.e. $\frac{a_{LC}}{a_{KC}} = \frac{4}{1} > \frac{1}{1} = \frac{a_{LF}}{a_{KF}}$. Thus, the Stolper-Samuelson theorem asserts that:

$$\hat{w} > \hat{p}_C = 50\% > \hat{p}_F = 0 > \hat{r}. \qquad \textbf{(2)}$$

Thus, labor gains unambiguously and capital loses unambiguously from this price change. With Leontief production functions, equations (1) are linear equations in w and r, so they can be solved simultaneously for any values of P_F and P_C. That is:

$$\begin{aligned} P_F &= 4 = a_{LF}w + a_{KF}r = (1\times w) + (1\times r) \\ P_C &= 15 = a_{LC}w + a_{KC}r = (4\times w) + (1\times r) \end{aligned} \qquad \textbf{(3)}$$

One way to do this is is to transform equations (3) into an explicit relationship between w

and r, plot both lines, and find the wage and rental that satisfy these two equations. Alternatively, one can solve the equations directly by substitution or using Cramer's rule. To do the latter, note that (3) can be rewritten as:

$$\begin{bmatrix} P_F \\ P_C \end{bmatrix} = \begin{bmatrix} a_{LF} & a_{KF} \\ a_{LC} & a_{KC} \end{bmatrix} \begin{bmatrix} w \\ r \end{bmatrix} \Rightarrow \begin{bmatrix} 4 \\ 15 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} w \\ r \end{bmatrix}.$$

The determinant of the matrix A = $[a_{ij}]$, $|A| = -3$, $|A_w| = -11$, $|A_r| = -1$. Thus:

$w = |A_w| / |A| = 11/3$;
$r = |A_x| / |A| = 1/3$.

These can be compared to $w = 2$ and $r = 2$ to give the ranking, given in (2), derived from the Stolper-Samuelson theorem:

$$\hat{w} = .83 > \hat{p}_C = .5 > \hat{p}_F = 0 > \hat{r} = -.83. \quad \textbf{(5)}$$

2. Retain the assumptions about technology in problem 1: $a_{LC} = 4$, $a_{KC} = 1$, $a_{LF} = 1$, $a_{KF} = 1$.

a. Draw a diagram with capital on the vertical axis and labor on the horizontal axis. Draw a ray through the origin with a slope of unity, and show how outputs of food and clothing can be measured along this ray. Draw a flatter ray, with a slope of 1/4, and show how outputs of clothing can be measured along this ray.

The rays with slope 1 and 1/4 are the fixed expansion paths for these Leontief technologies. Because these are constant returns to scale technologies, once we decide on a unit, all outputs are measured, in terms of those units, along the expansion paths.

b. Suppose the economy possesses 1000 units of labor. Find the full-employment output of each good if the capital stock is 500 units.

As in the textbook, we use the full-employment conditions to answer this question.

$$\begin{aligned} \bar{L} &= a_{LF} y_F + a_{LC} y_C \\ \bar{K} &= a_{KF} y_F + a_{KC} y_C \end{aligned} \quad \textbf{(6)}$$

As in question 1, the fact that the technologies are Leontief, means the full-employment conditions are linear relations. We can write these in matrix form as:

$$\begin{bmatrix} \bar{L} \\ \\ \bar{K} \end{bmatrix} = \begin{bmatrix} a_{LF} & a_{LC} \\ \\ a_{KF} & a_{KC} \end{bmatrix} \begin{bmatrix} y_F \\ \\ y_C \end{bmatrix} \Rightarrow \begin{bmatrix} 1000 \\ \\ 500 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_F \\ \\ y_C \end{bmatrix}. \qquad \textbf{(7)}$$

Now we just use Cramer's rule again, this time on A^T. $|A^T| = -3$, $|A^T{}_L| = -1000$, $|A^T{}_K| = -500$.

$y_F = |A^T{}_L|/|A^T| = 1000/3$,
$y_C = |A^T{}_C|/|A^T| = 500/3$.

c. Find the lowest and highest capital stocks that still allow full employment of both factors.

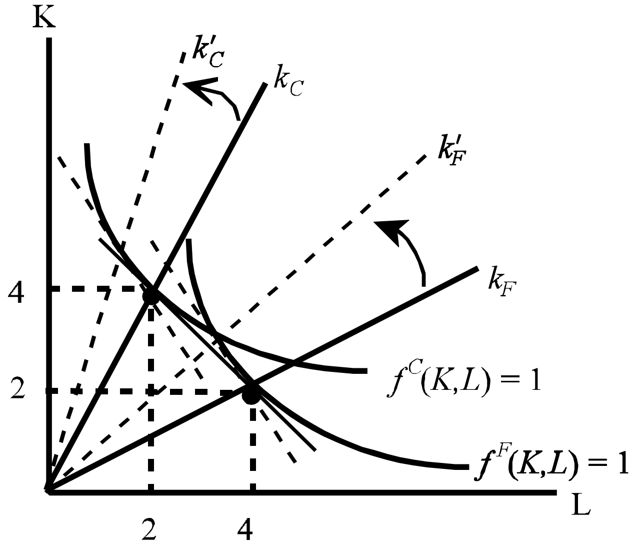
The lowest and highest capital stocks consistent with full-employment are determined by specialization in the *K*-intensive good and the *L*-intensive good. Thus, the highest capital stock consistent with full-employment involves specialization in the *K*-intensive good (food), which uses *K* and *L* in a 1/1 ratio. Since there are 1000 units of *L*, specialization in food would use 1000 units of *K*. The lowest capital stock consistent with full-employment involves specialization in clothing, which uses *K* and *L* in a 1/4 ratio. This means that specialization in *F* would require 250 units of *K*.

d. Draw the transformation schedule for each of the cases in 2a and 2b.

Workbook problems, 1, 4-6

1. *Isoquants and Factor Intensity*:

a. In a diagram with capital on the vertical axis and labor on the horizontal axis, draw smoothly bowed-in isoquants which satisfy the following two conditions:



i. Unit isoquants pass through the points given by:

$a_{KC} = 4$ $a_{KF} = 2$

$a_{LC} = 2$ $a_{LF} = 4$

and the wage-rental ratio is the same in the two industries at these points.

ii. No factor-intensity reversals.

b. Which Industry uses labor relatively intenstively?

From the data given in the question we have: $\frac{a_{LF}}{a_{KF}} = \frac{4}{2} > \frac{a_{LC}}{a_{KC}} = \frac{2}{4}$, so food is labor intensive in the initial equilibrium. However, by the no factor-intensity reversal assumption, we know that this must be true at every w/r, so food is the labor intensive industry.

c. In your diagram, show that for any change in the wage-rental ratio, the industry you chose in part (b) is always the labor-intensive industry. Show also that when the wage-rental ratio rises, both industries choose more capital-intensive techniques.

As long as the unit isoquants intersect once, and only once, given the assumption that they are constant returns to scale and permit smooth substitution between K and L, for any common w/r, $k_C > k_F$. In particular, as illustrated, an increase in the common w/r results in both industries becoming more K-intensive. This makes sense, since K and L can be substituted, when the relative price of L rises rational entrepreneurs will seek to substitute K for the now more expensive L.

4. *Changing factor use*: Consider two countries with the technology shown in the two figures and

assume that the (initial) equilibrium world price is $p = P_C / P_F$.

a. Suppose the world price of clothing rises by 10% while the price of food remains constant. Draw in the new production point on the production possibilities frontier.

An increase in P_C with P_F held constant will raise the relative price of C (i.e. p rises, say $p' > p$). At the initial equilibrium, $p' >$ MRT so the relative price of C exceeds its opportunity cost in production, leading to increased C output. The new equilibrium will occur at a tangency between the PPF and a national income line with slope p'–i.e. where MRT = p'.

b. Applying the Stolper-Samuelson theorem, what should happen to the wage-rental ratio? Draw in the tangencies along the unit isoquants at the new wage rental ratio. What happens to the capital-labor ratio in each industry?

The isoquant diagram identifies C as the K-intensive good. Thus, from the Stolper-Samuelson theorem, we have:

$$\hat{r} > \hat{p}_C = .1 > \hat{p}_F = 0 > \hat{w}. \quad \textbf{(8)}$$

Let $\omega := {}^{w}\!/_{r}$ and denote the original wage-rental ratio ω and the new one ω′, the relationships in (8) imply that $\omega > \omega'$. Recalling that both sectors faced ω and now both face ω′, both sectors will optimal input choices will respond in the same direction. At the initial input bundle $\omega' <$ MRTS, that is, at new factor prices, the relative cost of labor is less than the marginal physical product of labor relative to the marginal physical product of capital. Applying the arbitrage argument, entrepreneurs in both sectors will substitute labor for capital. Thus, the capital labor ratio will fall in both sectors.

c. Draw in the new allocation of labor and capital between the two industries in the production box. (Make sure that the slope along the unit isoquants is the same in both diagrams.) Draw in the capital-labor ratios at the new equilibrium point and the contract curve.

In addition to getting the slopes right between diagrams, the essential thing is to find a qualitatively correct new point on the efficient locus ("contract curve"). That is, the new expansion paths, reflecting more L-intensive production in both sectors, must intersect at a point of tangency between a higher valued C isoquant and a lower valued F isoquant. It is the slope of this tangency that needs to be the same as that of the slopes in the unit isoquant diagram from part (b).

5. *Comparative Advantage in the Heckscher-Ohlin Model*: The amount of capital and labor required to produce one unit of food and one unit of clothing are given by:

$$a_{KC} = 1 \qquad a_{KF} = 2$$
$$a_{LC} = 3 \qquad a_{LF} = 2$$

Assume that the techniques of production are invariant to the wage-rental ratio.

a. Suppose the home country is endowed with 160 units of labor and 100 units of capital.

i. Draw the labor and capital constraints in output space. (Put y_C on the horizontal axis and y_F on the vertical axis.) Identify the slopes of the two constraints.

Start with the full-employment conditions:

$$\begin{aligned} \bar{L} &= 160 = a_{LF}y_F + a_{LC}y_C = 2y_F + 3y_C \\ \bar{K} &= 100 = a_{KF}y_F + a_{KC}y_C = 2y_F + 1y_C \end{aligned} \quad \textbf{(9)}$$

Transform the equations so that they are in slope-intercept form:

$$\begin{aligned} y_F &= \frac{\bar{L}}{a_{LF}} - \frac{a_{LC}}{a_{LF}} y_C \Rightarrow y_F = \frac{160}{2} - \frac{3}{2} y_C \\ y_F &= \frac{\bar{K}}{a_{KF}} - \frac{a_{KC}}{a_{KF}} y_C \Rightarrow y_F = \frac{100}{2} - \frac{1}{2} y_C \end{aligned} \quad \textbf{(10)}$$

These are easily plotted in y_C -y_F space. Note that the the labor constraint has a higher intercept (80 > 50) and a steeper slope (3/2 > ½). Recall from the text that the relative slopes of these two constraints are determined by the factor-intensity assumption, i.e.:

$$\frac{a_{KC}}{a_{LC}} < \frac{a_{KF}}{a_{LF}} \Rightarrow \frac{a_{KC}}{a_{KF}} < \frac{a_{LC}}{a_{LF}} \quad \textbf{(11)}$$

ii. At full employment, how much clothing and food does the home country produce?

As in the previous questions, we can use Cramer's rule to solve for outputs. Thus, write the system from (9) in matrix form to get:

$$\begin{bmatrix} \bar{L} \\ \\ \bar{K} \end{bmatrix} = \begin{bmatrix} a_{LF} & a_{LC} \\ \\ a_{KF} & a_{KC} \end{bmatrix} \begin{bmatrix} y_F \\ \\ y_C \end{bmatrix} \Rightarrow \begin{bmatrix} 160 \\ \\ 100 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y_F \\ \\ y_C \end{bmatrix}. \quad \textbf{(12)}$$

$|A^T| = -4$, $|A^T{}_L| = -140$, $|A^T{}_K| = -120$.

$y_F = |A^T{}_L|/|A^T| = 140/4 = 35$,
$y_C = |A^T{}_C|/|A^T| = 120/4 = 30$.

b. Suppose the Foreign country is endowed with 120 units of labor and 80 units of capital.

i. Draw the labor and capital constraints in output space. (Put y_C on the horizontal axis and y_F on the vertical axis.) Identify the slopes of the two constraints.

Again, we start with the full-employment conditions:

$$\begin{aligned} \bar{L}* &= 120 = a^*_{LF} y^*_F + a^*_{LC} y^*_C = 2y^*_F + 3y^*_C \\ \bar{K}* &= 80 = a^*_{KF} y^*_F + a^*_{KC} y^*_C = 2y^*_F + 1y^*_C \end{aligned} \quad \textbf{(13)}$$

Transform the equations so that they are in slope-intercept form:

$$\begin{aligned} y^*_F &= \frac{\bar{L}}{a^*_{LF}} - \frac{a^*_{LC}}{a^*_{LF}} y^*_C \Rightarrow y^*_F = \frac{120}{4} - \frac{3}{2} y^*_C \\ y^*_F &= \frac{\bar{K}}{a^*_{KF}} - \frac{a^*_{KC}}{a^*_{KF}} y^*_C \Rightarrow y^*_F = \frac{80}{2} - \frac{1}{2} y^*_C \end{aligned} \quad \textbf{(14)}$$

These are easily plotted in y_C -y_F space. Note that the the capital constraint has a higher intercept (40 > 30) and a steeper slope (2 > ½). Note, because the technical coefficients are the same in Home and Foreign, the slopes of the *K* and *L* constraints must be the same. However, because the endowments differ, the intercepts differ.

ii. At full employment, how much clothing and food does the home country produce?

As in the previous questions, we can use Cramer's rule to solve for outputs. Thus, write the system from (13) in matrix form to get:

$$\begin{bmatrix} \overline{L}* \\ \\ \overline{K}* \end{bmatrix} = \begin{bmatrix} a_{LF} & a_{LC} \\ \\ a_{KF} & a_{KC} \end{bmatrix} \begin{bmatrix} y_F^* \\ \\ y_C^* \end{bmatrix} \Rightarrow \begin{bmatrix} 120 \\ \\ 80 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y_F^* \\ \\ y_C^* \end{bmatrix}. \qquad \textbf{(15)}$$

$|A^T| = -4$, $|A^{T}*_L| = -120$, $|A^{T}*_K| = -80$.

$y_F* = |A^{T}*_L|/|A^T| = 120/4 = 30$,
$y_C* = |A^{T}*_C|/|A^T| = 80/4 = 20$.

c. Compare the outputs of the two countries. What theorem would have predicted this pattern of production?

Under the assumptions of this question, the Rybczynski theorem predicts that the country that is more abundantly endowed with L will produce relatively more of the good that whose production is L-intensive. We have already seen, in (11), that clothing is L-intensive. To find the relatively L-abundant country we need to check:

$$\frac{\overline{L}}{\overline{K}} > \frac{\overline{L}*}{\overline{K}*} \Leftrightarrow \frac{160}{100} > \frac{120}{80} \quad \text{(i.e. } 1.6 > 1.5\text{)}.$$

Thus, the Rybczynski theorem predicts that the Home country will produce relatively more clothing and less food than the Foreign country. Since

$$\frac{30}{35} > \frac{20}{30},$$

the prediction is borne out by the analysis.

d. Assume that the two countries have the same tastes.

i. Which country will export food? Clothing?

With identical, homothetic preferences, at common prices both countries will consume food and clothing in the same proportions. Home is producing relative more clothing than Foreign at common prices, the only way for this to be true is if Home exports clothing and Foreign exports food.

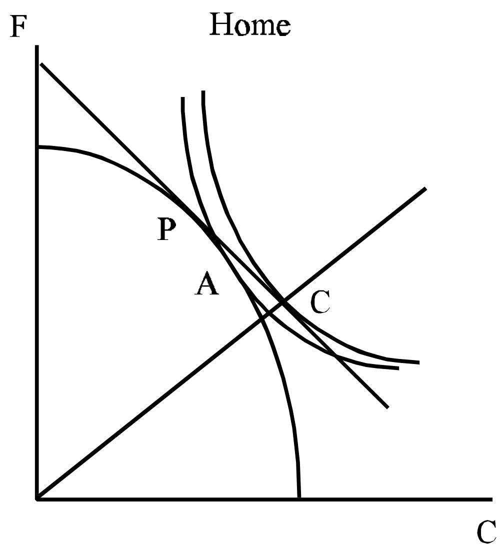
ii. Could you have predicted this result on the basis of factor endowments and the factor-intensities of the two industries? What theorem are you using to make this prediction?

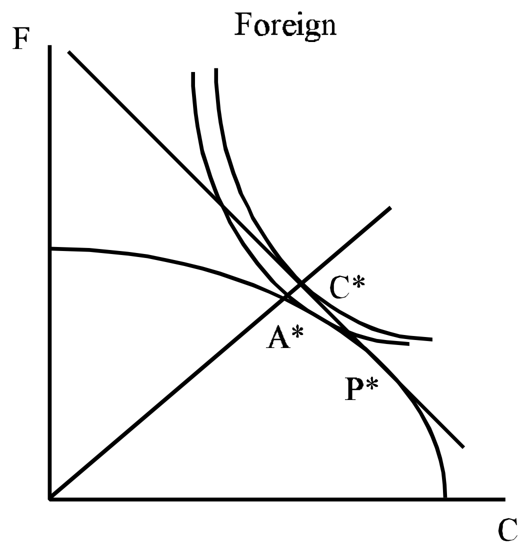
The HO theorem predicts that a country will have a comparative advantage, and thus (by the law of comparative advantage) export, the good whose production uses the country's abundant factor intensively. Because the Home country is relatively *L*-abundant, and clothing is relatively *L*-intensive, the HO theorem predicts that Home will export clothing.

6. *Relative Factor-Abundance and Trade Patterns*: The production possibilities frontiers below are drawn under the assumption that the Home country is endowed with a higher proportion of capital to labor than the Foreign country. Capital is used relatively intensively in the production of food. Preferences in the two countries are identical and homothetic.

a. Suppose these countries begin to trade. Draw in the new production and consumption points for each country.

Note that, in these drawings, A (A*) is the autarky equilibrium point, while P (P*) and C (C*) are the post trade production and consumption points for the Home (Foreign) country.





b. What happened to the pattern of production? Have the countries become more or less specialized as a result of trade?

If both countries are large, so post-trade world relative price settles strictly between the two autarky prices, both countries become more specialized in production of the commodity in which they have a comparative advantage. Thus, their patterns of production become more dissimilar. Even if one country is large and the other small, because the small country will become more specialized, the countries become more dissimilar.

c. What happened to the pattern of consumption? Have the bundles in the Home and foreign countries become more or less similar as a result of trade?

With identical, homothetic preferences, if trade equalizes commodity prices, the *relative* amounts of food and clothing in consumption must be identical. In the diagram, this should be reflected in common slopes for the income-consumption lines in the two diagrams. If countries happened to have the same post-trade incomes, the bundles consumed would be identical.

d. What do you think has happened to the wage-rental ratio in each country?

Since trade equalizes commodity prices, the *Factor-price equalization theorem* tells us that the wage-rental ratios in the two countries have become more similar. If both countries remain unspecialized in the new equilibrium, as they do in the above diagram, the wage-rental ratios must be identical (i.e. equal).