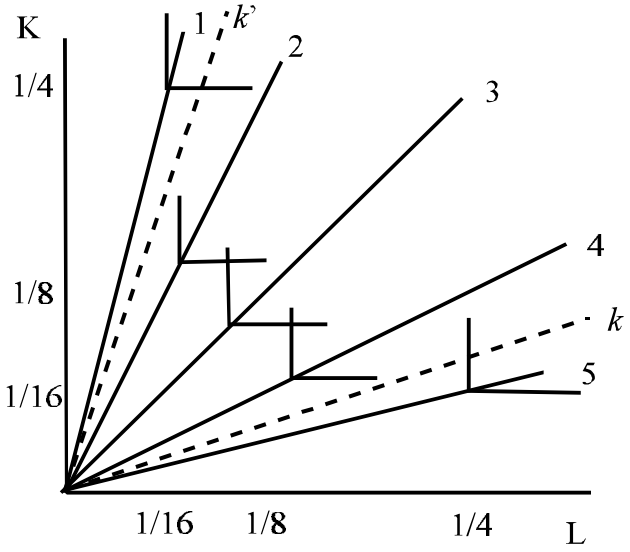
Homework #7: Answers

Text questions, Chapter 8, problems 1-3.

1. Assume that input-output coefficients are fixed. The table shows capital requirements per unit output (a_{kj}) and labor requirements per unit output (a_{Lj}) to produce one unit each of commodities 1-5. Also shown are prevailing prices for each commodity.

	Commodity				
	1	2	3	4	5
a_{kj}	4	2	1	1	1
a_{Lj}	1	1	1	2	4
Price	\$16	\$14	\$10	\$14	\$16

a. If the economy has a labor/capital endowment ratio of 3:1, what does it produce? What are the wage rate and rents on capital?



Construct the *unit value isoquants* for each of these sectors and plot the endowment ratio (denoted *k* in the diagram). From the picture, it is easy to see that the economy will produce goods 4 and 5. To find the wage and rentals consistent with these, use the zero profit conditions. In matrix form, these are:

$$\begin{bmatrix} P_4 \\ \\ P_5 \end{bmatrix} = \begin{bmatrix} a_{L4} & a_{K4} \\ \\ a_{L5} & a_{K5} \end{bmatrix} \begin{bmatrix} w \\ \\ r \end{bmatrix} \Rightarrow \begin{bmatrix} 14 \\ \\ 16 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ \\ 4 & 1 \end{bmatrix} \begin{bmatrix} w \\ \\ r \end{bmatrix}.$$

The determinant of the matrix A = $[a_{ij}]$, $|A| = -2$, $|A_w| = -2$, $|A_r| = -24$. Thus:

$w = |A_w| / |A| = 2/2 = 1;$
$r = |A_r| / |A| = 24/2 = 12.$

b. If the world price of commodity 1 should triple, would there be any change in this country's production pattern? Factor Prices? Real income?

If the world price of commodity 1 triples, from \$16 to \$48, the unit value isoquant for commodity 1 would shift toward the origin sufficiently far that only goods 1 and 5 are now producible. We can solve for the new factor prices as above:

$$\begin{bmatrix} P_1 \\ \\ P_5 \end{bmatrix} = \begin{bmatrix} a_{L1} & a_{K1} \\ \\ a_{L5} & a_{K5} \end{bmatrix} \begin{bmatrix} w \\ \\ r \end{bmatrix} \Rightarrow \begin{bmatrix} 48 \\ \\ 16 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ \\ 4 & 1 \end{bmatrix} \begin{bmatrix} w \\ \\ r \end{bmatrix}.$$

The determinant of the matrix A = $[a_{ij}]$, $|A| = -15$, $|A_w| = -16$, $|A_r| = -176$. Thus:

$w = |A_w| / |A| = 16/15;$
$r = |A_r| / |A| = 176/15.$

c. At the initial prices shown in the table, how would factor prices differ in an economy with the same technology but a capital/labor ratio of 3:1?

This endowment ratio is shown by the line k' in the diagram. Under initial prices, with endowment ratio k', the economy would produce goods 1 and 2. Using the method from the previous two parts (i.e. Cramer's rule) we can solve for $w = 12$ and $r = 1$.

d. For the economy with the original labor/capital ratio of 3:1, how would production patterns and factor prices change if commodity 3's price on world markets should rise by 40 percent?

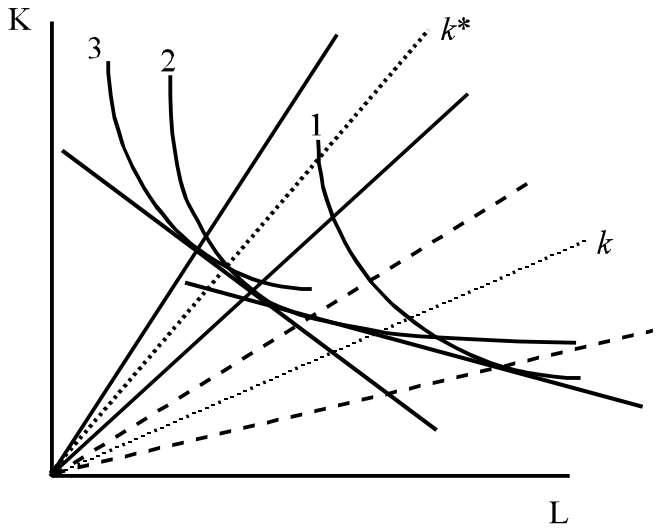
A 40 percent increase in the price of good 3, with endowment ratio k, will result in goods 3 and 5 being produced. Using the zero profit conditions, we can find $w = 2/3$ and $r = 13$ 1/3.

e. Describe the pattern of trade for an economy with a capital/labor endowment ratio of 1:1.

For an economy with a K/L ratio of 1:1, the economy will specialize in the production of commodity 3.

2. Using Figures 8.4 and 8.5, show how an increase in the world price of commodity 2 would affect real wages in two countries sharing the same technology, both of them producing commodity 2, but with the home country having an endowment ratio of 0I (Figure 8.5) and the foreign country with endowment ratio 0M.

The essential fact from figure 8.5 is that the Home country has an endowment ratio that permits production of goods 1 and 2, while the foreign country has an endowment ratio that permits production of goods 2 and 3. That is, these endowments lie in different cones of diversification. The relevant part of figure 8.4 is reproduced below.



An increase in P_2 will shift the unit value isoquant for good 2 toward the origin. Let $\omega := w/r$ and $\omega^* = w^*/r^*$, and denote values following the price change with a "′". If the price change is sufficiently small that both countries continue to produce the same goods as before the price change, it must be the case that $\omega > \omega'$ and $\omega^* < \omega^{*\prime}$. Note that this could have been predicted from the Stolper-Samuelson theorem and the fact that good 2 is K-intensive for the Home country, but L-intensive for the Foreign country. If the price change is sufficiently large that both countries become specialized to production of good 2, the same relationships will hold (i.e. $\omega > \omega'$ and $\omega^* < \omega^{*\prime}$), but now this will follow from the relationship between the initial tangencies between unit isocost and unit value isoquant and the slope of the unit value isoquant at the intersection with the endowment rays.

3. In figure 8.2 points H' and F' show each country devoting the same resources to the differentiated clothing sector as it did in autarky. Suppose that the Home country is relatively labor-abundant and clothing is labor-intensive relative to food.

a. How would this alter the number of firms devoted to clothing in each country?

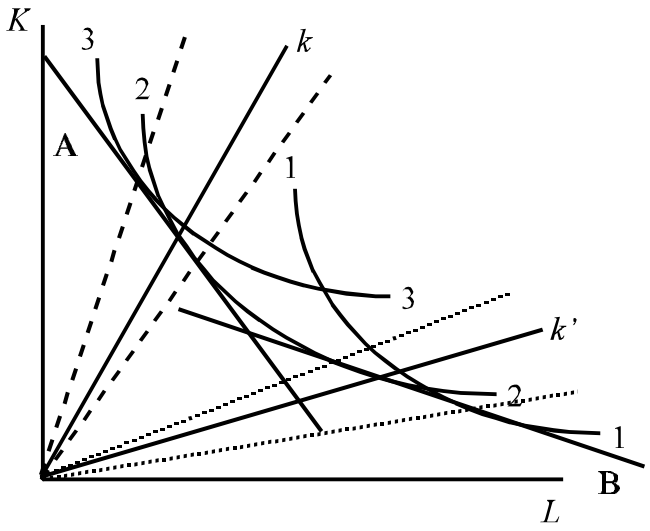
The fact that the Home country is *L*-abundant and clothing production *L*-intensive means that, with identical homothetic preferences, production costs in autarky will be lower in the Home country. Thus, there will be more firms operating in the Home country, *ceteris paribus*, under this assumption than under the assumption identical endowments.

b. Could the Home country produce a wider variety of clothing with trade than the (larger) Foreign country?

This could result in there being more clothing producers in the Home country than abroad. Whether or not this actually happens would depend on the way that size and relative cost interact. Thus, for a sufficiently large difference in size and a sufficiently similar endowment, the larger (Foreign) country will continue to produce more varieties. On the other hand, for sufficiently similar size and sufficiently large difference in endowments, the Home country could produce more varieties.

Workbook problems, 1, 2, and 6

1. *Unit-value Isoquants, Production Pattern, and Factor Prices*: The isoquants drawn below show the amounts of capital and labor required to produc $1 worth of each of the goods.



a. Suppose the endowment of this economy is shown by the $k = K/L$ ray. What good(s) will be produced?

Recall that these are unit value isoquants, defined relative to equilibrium commodity, and thus factor, prices. Since k lies in the cone determined by a tangency between isocost A and isoquants 3 and 2, those goods will be produced.

b. Suppose that there is an influs of workers and the capital/labor ratio falls to $k' = K/L'$. Now what will be produced?

Because k' falls in the cone determined by the tangency between isocost B and the 2 and 1 isoquants, those 2 goods will now be produced.

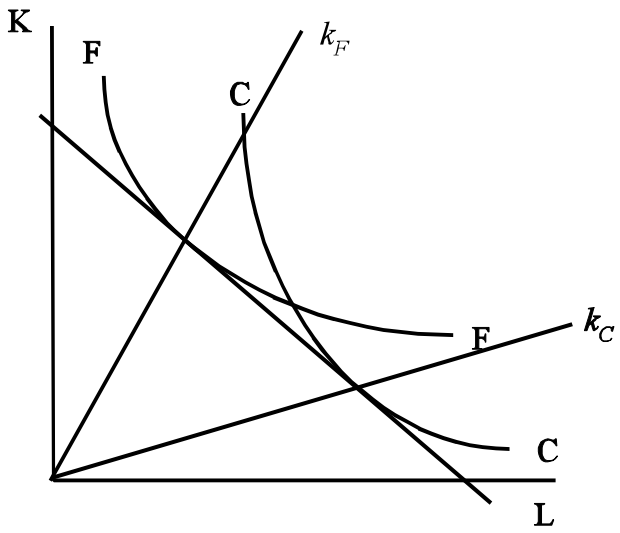
c. What happened to the wage/rental ratio as the k ratio fell?

As k moved within it's original cone, there was no change in the w/r ratio. Once it passed outside the cone, the w/r ratio falls smoothly along the 2 isoquant (as the w/r ratio is being determined by MRTS in 2 production). Finally, once it passes into the second cone, the w/r ratio is fixed by the slope of the common tangent–i.e. the slope of isocost 2.

d. What happens to the wage/rental ratio as compared to (c) if the price of good 1 doubles?

The doubling of the price of good 1 will cause the unit value isoquant to shift halfway to the origin. This will cause the equilibrium *w*/*r* ratio to rise. Note, this change may cause good 2 to go out of production, first being replaced by good 3, and, if the shift is proportionally large enough, to specialization in good 1.

2. *The Factor-Price Equalization Theorem Once Again*: Consider a two-country world, where both countries have the same technology for producing clothing and food as illustrated by the following unit-value isoquants:



a. What is the range of factor endowment ratios that will yield factor price equalization? Indicate this range in the diagram.

Any endowment in the cone of diversification is consistent with factor-price equalization.

b. Does this range depend upon commodity prices?

Yes. Recall that these are unit *value* isoquants. Thus, their location, and the location of the tangencies that define k_F and k_C are determined by commodity prices. Thus, different commodity prices would produce a different cone.

6. *Love of Variety versus Indifference to Variety*: The Home and Foreign countries each possess the same technology for producing blue and red schmoos. Production of a single schmoo (blue or red) requires 30 units of capital and 30 units of labor, while the second schmoo requires only an additional 10 units of capital and 10 units of labor. The third schmoo requires 5 units of capital and 5 units of labor extra. Each additional schmoo requires only one extra unit of capital and one extra unit of labor to be produced. If the first schmoo is red the other schmoos produced must be red to capture the increasing returns to scale.

a. Each country is endowed with 60 units of capital and 60 units of labor. Suppose consumers in each country demand blue and red schmoos in equal proportions. What will each country produce in autarky?

Because people prefer to consume red and blue schmoos in equal proportions, each country will produce both varieties in autarky. With identical technologies, this will involve the allocation of 30 K and 30 L to each variety. This permits production of 1 red and 1 blue schmoo in each country.

b. If consumers are indifferent about the color of schmoos, what does each country produce in autarky?

In this case each country will allocate all factors to the production of only one variety. In this case, a total of 18 units can be produced (i.e. 30 + 10 + 5 give the first three units, this leaves 15 additional units of K and L for a total of 3 + 15 = 18).

c. Now suppose that the two countries begin to trade and that consumers care about the color of the schmoo. What will each country produce in a trade equilibrium? What happens to the number of schmoos produced in each country relative to your answer to part (b)? What is the number of firms in each country?

In this case each country will specialize in a different variety of schmoo, producing 18 units, and trade. In this case, each country will consume 9 red and 9 blue schmoos. The total number produced in each country is the same as in part (b), the difference is that each country must produce a different variety. There are gains both from increased efficiency (relative to part a) and increased variety in consumption (relative to part b). Note that, because these are internal economies of scale, there must be 1 firm in each country.