Trade and Industrial Policy for an International Duopoly with U-Shaped Costs

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by Douglas R. Nelson and Shunichi O. Tsutsui
Economics Department
Tulane University
New Orleans, LA 70118-5698

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Sometime in the mid-1970s international competition began to emerge as a substantial public political issue in the US for the first time in at least half a century. Along with apparently intractable current account problems and competitive pressure on major US industrial sectors, GATT commitments to abjure direct protection made industrial policy an attractive issue. As sectoral policy became an important political issue, economists began to develop a body of theory that allowed direct engagement with the class of concerns raised by proponents of an activist industrial policy. The theory of strategic trade policy has addressed the distinction between trade and industrial policy primarily by focussing on tariffs and subsidies on the one hand and general production subsidies on the other (e.g. Eaton and Grossman, 1986). While this captures one important element of the distinction, this paper focuses on the distinction between policies oriented toward industrial structure and policy that takes such structure as fixed .[1] Specifically, we develop an extension of the Brander-Spencer (1985) model in which inter-firm competition in a third market takes place in two stages: in stage one firms choose capacity; and in stage two they choose quantites. We will refer to policies that attempt to affect the capacity choice as "industrial policy" and policies that attempt to affect the choice of output/exports as "trade policy".

The basic Brander-Spencer model, and virtually all of the extensions that have appeared in the literature consider a very simple cost structure--usually constant marginal cost with a fixed cost. To the extent that this line of research does consider capacity decisions, it takes the Kreps-Schienkman (1983) form of fixed capacity choice followed by price competition (e.g. Ben-Zvi and Helpman, 1992).

[1]It is worth noting that one of the earliest papers on strategic trade policy, Auquier and Caves (1979) focussed directly on the question of endogenous determination of industrial structure. In their case this was taken to mean the determination of "competitiveness" via anti-trust policy, while in this paper we are concerned with physical capacity.

In this paper we are interested in the implications of *ex post* flexibility for trade and industrial policy, so we choose a simple specification with U-shaped short-run average costs.

The next section spells out our model and derives baseline results for the case of a single monopolist selling to a foreign market. This is followed by analysis of Cournot competion between national monopolists with u-shaped average costs--our simple extension of the Brander-Spencer model. In this context we examine the effect of industrial policy (subsidies to scale) and trade policy (subsidies to exports and quantity controls on exports). The paper concludes with a comparison of the policy regimes under varying degrees of production flexibility.

I. Framework and Monopoly Baseline

Following the approach introduced by Brander and Spencer, we abstract from domestic market considerations by considering competition between a pair of national monopolists in a third market. With identical products and linear demand, denoting sales by the Home and Foreign monopolists as q and q^*, we can write inverse demand as $p(Q) = \alpha - bQ$ where $Q = q + q^*$. Because our main concern is with flexibility following the capacity decision, we assume that the Home and Foreign firms produce perfect substitutes subject to the following U-Shaped cost function:

$$C(q,x) = mq + c(q - x)^2,$$

where x denotes the plant size of minimum efficient scale (MES) chosen by the firm.[2] Since

$$MC = m + 2c(q - x) \text{ and } AC = m + cq - 2cx + \frac{cx^2}{q},$$

the AC has a minimum m at $q = x$. It is clear that as long as the feasible plant size is bounded above, then by assuming a sufficiently large m, we can ensure that MC is always positive for any $q \geq 0$. Note

[2] See appendix 1 for a discussion of this cost function.

that the minimum cost of production for every plant size is constant and equal to m.

Now consider the case in which the international market is monoplized by the Home firm. Let q_M and x_M denote the monopolist's output and plant size. As usual, the monopoly case serves as a benchmark which gives the theoretically maximum profits the Home firm can make in the Brander-Spencer setting. Given plant size x_M, the monopolist faces the following problem

$$\max_{q_m} \Pi_M = q_M(\alpha - bq_M) - mq_M - c(q_M - x_M)^2 = q_M(a - bq_M) - c(q_M - x_M)^2,$$

where we have substituted $a = \alpha - m > 0$. From the first-order condition for this problem we get

$$\bar{q}_M = \frac{a + 2cx_M}{2(b + c)}.$$

We can substitute this into the profit function to get

$$\Pi_M = (b + c)\bar{q}_M^2 - cx_M^2,$$

from which we have that

$$\frac{\partial \Pi_M}{\partial x_M} = 2(b + c)\bar{q}_M \frac{2c}{2(b + c)} - 2cx_M = 2c(\bar{q}_M - x_M) = 0.$$

Using this, we obtain

$$x_M = q_M = \frac{a}{2b} \text{ and } \Pi_M = \frac{a^2}{4b}. \qquad \textbf{(0)}$$

Note that in the absence of strategic interaction, the monopolist chooses the plant size at which the short-run average cost is minimized.

Equilibrium Scale, Output and Profits under Four Policy Regimes

Now consider the following simple extension of the Brander-Spencer model: two national monopolists compete in a third market; but now there are two stages--firms choose capacity in the first stage and output/exports in the second stage. In this section we review the equilibrium scale {x,x*}, output {q,q*}and profits {Π,Π*} under each of the following regimes: non-intervention; trade intervention by export subsidy; trade intervention by quantity control; and industrial policy.[3] These results are summarized in table 1. Follwing the standard approach, we take the Home government to be active and the Foreign government to be passive.

Standard backward-induction from quantity-competition to optimal scale provides the Nash equilibrium values of scale, output and profit given in table 1

Comparing Policy Regimes with Varying Flexibility in Production

In the previous sections we have derived the equilibria of two-stage economic competition under a variety of policy regimes: no intervention; trade policy (both quantity control and subsidy); and industrial policy (subsidies to capacity). In this section we compare the effectiveness of these policy regimes for industries characterized by varying degress of flexibility in production. Since the return to an uncontested monopolist gives the theoretical maximum welfare for the Home country, we will measure the effectiveness of policy as a fraction of the monopolistc outcome. This measure has the particular virtue that it is free from scale and permits easy comparison. Thus, letting the subscript M refer to the uncontested monopoly outcome, we define the index $G = W_j/W_M$, where j is either: n = non-intervention; q = quantity control; s = subsidy; or i = industrial policy.[4] We define the various G

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[3]The derivations of these results, which are tedious but straightforward, can be found in appendix 2.

[4]Recall that for the cases of monopoly, non-intervention, and quantity control, none of which involve a subsidy, $W_j = \Pi_j$, in the cases involving a subsidy, national welfare is given by the monopoly return net of the subsidy.

indices as follows:

$$G_n = \frac{W_n}{W_M},\ G_q = \frac{W_q}{W_M},\ G_s = \frac{W_s}{W_M},\ G_i = \frac{W_i}{W_M}.$$

From the derived values of the W_j in equations (0),(1), (3), and (5), we can see that the G_j are functions only of t ($= c/b$). Note that with the demand curve held constant, t varies directly with c.

First consider the case of maximum flexibility (i.e. $c = 0$). In this case the choice of plant size is irrelevant because it does not bind on the choices made in the quantity-competition stage. In this case, it is well-known that both quantity control and subsidy policies yield identical outcomes (since no uncertainty is present).

Proposition 1: If $c = 0$, $G_n = G_i = \frac{4}{9}$ (≈ 0.44) and $G_q = G_s = \frac{1}{2}$ ($= .50$).

Note that industrial policy is ineffective since plant size does not affect the firms quantity decisions, thus the optimal policy is zero subsidy and the outcome is identical to the non-intervention case. Proposition 1 says that either form of trade policy raises the country's welfare relative to *laissez faire* (or industrial policy).

In proposition 2 we show that production inflexibility affects policy effectiveness. Consider the other extreme case where firms are completely inflexible (i.e. $c \to \infty$). In such a case, it is no longer meaningful to refer to a quantity decision since the choice of capacity completely determines the quantity decision. Thus, we would expect that policies intended to affect the quantity decision (i.e. trade policies) would lose their effectiveness, while policies aimed at the capacity decision (i.e. industrial policy) would be effective. This is indeed what the next proposition reports.

Proposition 2: As $c \to \infty$, $G_n \to \frac{4}{9}$, $G_q \to \frac{4}{9}$, $G_s \to \frac{4}{9}$, and $G_i \to \frac{1}{2}$.

Propositions 1 and 2 make it clear that optimal policy with either complete flexibility or complete inflexibility of production can reproduce the welfare results of the standard Brander-Spencer model, though it is important to recognize that in our model policy success depends on adopting the correct policy. This raises the question of what happens between these two obviously extreme cases. It seems likely that between the points of perfect flexibility and perfect inflexibility there would be a switching point where trade policies become less effective than industrial policy. Figure 1 shows the values of the various G-indices for varying levels of t. The upper envelope of these curves shows trade policy (quantity control at low values of c, and then subsidies at intermediate values) giving way to industrial policy as the most effective policy at high levels of c (specifically, at $t = 3.8$). In addition, it is worthwhile to note that:

1. Industrial policy is always more effective than non-intervention;

2. Quantity control is always more effective than non-intervention;

3. The export/production subsidy is less effective than quantity control when t is small but not zero, while this is reversed when t is large; and

4. The export/production subsidy is less effective even than the non-intervention policy when t is small but becomes more effective when t is large.

Conclusions 1 and 2 are unsurprising and consistent with standard conclusions in strategic trade theory: the capacity to affect the Home firm's quantity prior to the quantity competition permits policy to improve on the laissez faire outcome. The results on the export subsidy, however, are initially unexpected. The key to understanding this result is the recognition that, with a subsidy, the firm makes profit both from sales in the third market at above the cost of production and from subsidy payments made by the Home government. We have already seen that our model is characterized by equilibrium excess capacity without intervention. That is, firms are producing with decreasing costs in equilibrium. To extract additional subsidy from the government, the firm chooses an even larger plant size.[5] The logic is straightforward: in addition to the gain from shifting profit,

[5] This is easily seen as a form of revenue-seeking activity of the form studied in general equilibrium by Bhagwati (1982) and Bhagwati and Srinivasan (1980).

since the firm is on the decreasing cost portion of its cost curve there is an additional source of gain from trade activism. The firm chooses its plant size in expectation of the subsidy. The more flexible is the firm's technology, the larger must be the deviation from the laissez-faire plant size to extract subsidies based on returns to scale. However, since this ultimatley commits the firm to greater export sales and lower prices, the firm must weigh the gain from additional subsidy against the loss from increased market saturation. Figures 2 and 3 to illustrate this discussion. Figure 2 shows the equilibrium plant sizes under the four regulatory regimes as flexibility varies. It is easy to see that, except at $c = 0$, plant size increases monotonically with flexibility, but in figure 3 we see output rising to a peak and then declining with flexibility. Even though scale is very large, the effect of large output on the equilibrium price disciplines the firm's output decision. Thus, in the range of moderately high flexibility, the existence of a subsidy policy induces maximum inefficiency and generates the large drop in social welfare (G) that we observe in figure 1.

One way of interpreting these results is as additional evidence of the sensitivity of policy recommendations based on simple models of imperfectly competitive markets. That is, it is important to recognize that all of the standard caveats that apply to the Brander-Spencer results apply here as well (see, e.g. Krugman, 1987). Specifically, even in the simple Brander-Spencer context, both informational and political constraints render claims about obvious gains from trade activism extremely doubtful. While our results offer no additional insight with respect to the political constraints, the additional complexity implied by the importance of both optimal scale and flexibility suggest further difficulties in implementing a welfare-improving industrial/trade policy.

Table 1

	Non-Intevention (ni)	Quantity Control	Export Subsidy[6]	Industrial Policy[7]
x	$\frac{4(1+t)^2}{9+22t+12t^2}\cdot\frac{a}{b}$	$\frac{2(1+t)^2}{(1+2t))(4+3t)}\cdot\frac{a}{b}$	$\frac{4(1+t)^4(1+2t)}{4+27t+76t^2+118t^3+88t^4+24t^5}\cdot\frac{a}{b}$	$x_{ni}+\frac{(3+2t)(1+2t)(9+32t+40t^2+16t^3)h}{2t(3+6t+4t^2)(9+22t+12t^2)}$
q	$\frac{(3+2t)(1+2t)^2}{9+22t+12t^2}\cdot\frac{a}{b}$	$\frac{2(1+t)^2}{(1+2t))(4+3t)}\cdot\frac{a}{b}$	$\frac{1+4t+2t^2}{2(1+t)^2}x$	$\frac{(h-2btx)(3+2t)(1+2t)}{8t(1+t)^2 b}$
Π	$\frac{(1+t)(9+32t+40t^2+16t^3)}{(9+22t+12t^2)^2}\cdot\frac{a^2}{b}$	$\frac{2(1+t)^3}{(1+2t)(4+3t)^2}\cdot\frac{a^2}{b}$	$\frac{(1+4t+8t^2+4t^3)bx^2}{4(1+t)^3}$	$b(1+t)q^2-btx^2+hx$
x*	$\frac{4(1+t)^2}{9+22t+12t^2}\cdot\frac{a}{b}$	$\frac{(3+2t)}{2(4+3t)}\cdot\frac{a}{b}$	$\frac{(3+2t)(1+2t)(1+2t+6t^2+4t^3)}{2(4+27t+76t^2+118t^3+88t^4+24t^5)}\cdot\frac{a}{b}$	$x_{ni}^{*}-\frac{8t(1+t)^2(3+2t)(1+2t)h}{2t(3+6t+4t^2)(9+22t+12t^2)}$
q*	$\frac{(3+2t)(1+2t)^2}{9+22t+12t^2}\cdot\frac{a}{b}$	$\frac{1+4t+2t^2}{(1+2t)(4+3t)}\cdot\frac{a}{b}$	$\frac{2(1+4t+2t^2)}{(3+2t)(1+2t)}x^{*}$	$\frac{(3+2t)(1+2t)x^{*}}{4(1+t)^2}$
Π*	$\frac{(1+t)(9+32t+40t^2+16t^3)}{(9+22t+12t^2)^2}\cdot\frac{a^2}{b}$	$\frac{(4+27t+64t^2+56t^3+16t^4)bx^{*2}}{(3+2t)^2(1+2t)^2}$	$\frac{(1+27t+64t^2+56t^3+16t^4)bx^{*2}}{(3+2t)^2(1+2t)^2}$	$b(1+t)q^{*2}-btx^2+hx*$

[6] Where welfare is given by profit in the non-intervention and quantity control cases, in the subsidy case we need to calculate the net of subsidy measure of welfare. This is given by $W=\frac{(1+2t)(1+2t+10t^2+12t^3+4t^4)bx^2}{8(1+t)^5}$.

[7] In this case we solve for the optimal subsidy to scale, given by , and the welfare measure $W=\frac{(3+2t)^2(1+2t)^2(h-2btx)^2}{64t^2(1+t)^3 b}-btx^2$.

Appendix 1: Conditions on the Cost Function

As in Vives (1986) and Spencer and Brander (1992), we adopt this particular (quadratic) form for the cost function for its tractability. It should be noted that this is a valid cost function only when the following restrictions are satisfied. Let $X = (a\text{-}m)/b$, i.e. X measures the size of the market corresponding to zero profits. Suppose that one wants the cost function to be well-behaved for $x \in [0, nX]$, $n = 1, 2, \ldots$. This means that

$$MC = m + 2c(q - nX) \geq 0, q \geq 0.$$

Hence, we establish that

$$m - 2cnx = m - 2cn\frac{\alpha - m}{b} \geq 0 \Leftrightarrow \frac{\alpha}{m} \leq \frac{2cn + b}{2cn}.$$

Since $\alpha > m$, we obtain that

$$1 < \frac{\alpha}{m} \leq \frac{2cn + b}{2cn}.$$

As long as this condition is satisfied, the cost function in the paper is well-behaved. Note that since all the equilibrium plant sizes found in this paper are less than X, the above inequality is also sufficient to ensure that, at those equilibrium plant sizes, the MC is positive.

Appendix II: Derivation of Equilibrium Values of x, x*, q, q*, Π, and Π*

Cournot Competition between National Monopolists with U-shaped Average Costs

Now we consider a two-stage game between firms that will choose size (i.e. MES) in the first-stage and quantity supplied to the third market in the second stage. Following the standard practice, the firms will solve for the Cournot (Nash) equilibrium of the second stage and use that as input to the solution of the capacity game. Given the symmetry assumption standard in the literature, we need focus only on one of the firms. Denoting foreign magnitudes with a star, we solve the Home firm's problem. Thus, for given plant sizes {x, x*}, the Home firm's problem is

$$\max_q \Pi = q(\alpha - bq - bq^*) - mq - c(q - x)^2 = q(a - bq - bq^*) - c(q - x)^2,$$

From the first-order condition for this problem, we get

$$2(b + c)q + bq^* = a + 2cx.$$

Hence the equilibrium outputs in the quantity-setting stage are

$$\bar{q} = \frac{2(b + c)(a + 2cx) - b(a + 2cx^*)}{(3b + 2c)(b + 2c)}.$$

Now, to solve for the equilibrium plant sizes, we need to express the firms' profit functions in terms of plant sizes. Substituting the first-order condition in the profit function yields

$$\Pi = (b + c)\bar{q}^2 - cx^2,$$

from which we have

$$\frac{\partial \Pi}{\partial x} = 2(b + c)\bar{q}\frac{4c(b + c)}{(3b + 2c)(b + 2c)} - 2cx = 0.$$

Rewriting this yields

$$\hat{q} = \frac{(3b + 2c)(b + 2c)}{4(b + c)^2} x .$$

Note here that in equilibrium the firm chooses a plant size larger that the level of output it produces, that is: $(3b + 2c)/(4(b + c)^2) < 1$. This "excess capacity" reflects the competitive externality in the first-stage of the game. The standard Cournot externality continues to exist in the quantity competition stage. That is, while equilibrium output is below MES, it is above the output that maximizes joint profits. The solution values for the symmetric equilbirium are:

$$\begin{aligned} {}_i = x = x^* &= \frac{4(b + c)^2 a}{b(9b^2 + 22bc + 12c^2)} = \frac{4(1 + t)^2}{9 + 22t + 12t^2} \cdot \frac{a}{b} , \\ {}_i = q = q^* &= \frac{(3b + 2c)a}{b(9b^2 + 22bc + 12c^2)} = \frac{(3 + 2t)(1 + 2t)}{9 + 22t + 12t^2} \cdot \frac{a}{b} , \\ = \Pi = \Pi^* &= \frac{(b + c)(9b^3 + 32b^2c + 40bc^2 + 16c^3)a^2}{b(9b^2 + 22bc + 12c^2)^2} = \frac{(1 + t)(9 + 32t + 40t^2 + 16t^3)}{(9 + 22t + 12t^2)^2} . \end{aligned} \quad \textbf{(1)}$$

Note that we have set $t = c/b$. We can compare these values to the monopoly values

Trade Policy for a Duopoly with Output Flexibility

As suggested in the introductory remarks, we treat as trade policy those interventions that take the capacity choice as given.[8] That is, the government's move occurs between the firms' choice of capacity and their choice of quantity supplied to the third market. We will consider two forms of trade policy: quantity control; and export (i.e. output) subsidy.

[8]This distinction only strictly makes sense in the Brander-Spencer setup with all output going to the third market. Obviously, if there is domestic consumption a general subsidy to output is not strictly a trade policy.

Quantity Control

Under quantity control, the Home government sets the Home firm's export quantity (= output) before the Foreign firm chooses it's output. Because the government policy is fully credible, this makes it possible for the Home firm to act as a Stackelberg leader in the quantity-setting game. Since the Foreign firm acts as a Stackelberg follower, using the first-order condition of the Foreign firm and denoting values of Foreign variables with a star, its output decision rule is given by

$$\bar{q}^* = \frac{1}{2(b + c)}(a + 2cx^* - bq).$$

The Home government chooses the Home firm's quantity to maximize the Home firm's profits:

$$\max_q \Pi = q(a - bq - b\bar{q}^*(q)) - c(q - x)^2.$$

The first-order condition for the Home government's problem is

$$\frac{\partial \Pi}{\partial q} = a - 2bq - \frac{b(a + 2cx^*)}{2(b + c)} - 2c(q - x) = 0,$$

which, after substitution, yields the following equilibrium outputs in the quantity-setting stage:

$$\bar{q} = \frac{2(b + c)(a + 2cx) - b(a + 2cx^*)}{2(b^2 + 4bc + 2c^2)},$$

$$\hat{q}^* = \frac{(3b + 2c)(b + 2c)(a + 2cx) - 2b(b + c)(a + 2cx^*)}{4(b + c)(b^2 + 4bc + 2c^2)}.$$

To find the equilibrium plant sizes, we need to express the firms' profit functions in terms of plant sizes. Substituting the first-order conditions in the profit functions yields:

$$\Pi = (b+c)\,\bar{q}^2 - \frac{b^2}{2(b+c)}\,\bar{q}^2 - cx^2 = \frac{(b^2+4bc+2c^2)}{2(b+c)}\,\bar{q}^2 - cx^2,$$

$$\Pi^* = (b+c)\,\hat{q}^{*2} - cx^{*2}.$$

The national monopolists choose their plant sizes simultaneously and non-cooperatively. The first-order conditions are

$$\frac{\partial \Pi}{\partial x} = \frac{(b^2+4bc+2c^2)}{2(b+c)}\,2\bar{q}\,\frac{4c(b+c)}{2(b^2+4bc+2c^2)} - 2cx = 2c(\bar{q}-x) = 0,$$

$$\frac{\partial \Pi}{\partial x^*} = 2(b+c)\,\hat{q}^*\,\frac{2c(3b+2c)(b+2c)}{4(b+c)(b^2+4bc+2c^2)} - 2cx^* = \frac{c(3b+2c)(b+2c)}{b^2+4bc+2c^2}\,\hat{q}^* - 2cx^* = 0,$$

from which we get

$$\bar{q} = x,$$

$$\hat{q}^* = \frac{2(b^2+4bc+2c^2)}{(3b+2c)(b+2c)}\,x^*. \qquad \textbf{(2)}$$

Thus, under quantity control the Home firm is able to choose its capacity optimally in the sense that it produces its quantity at MES. The Foreign firm, however, continues to choose a larger than optimal plant size.

Using equations (2), we have that

$$\begin{bmatrix} 2b(b+2c) & 2bc \\ 4bc(b+c)(3b+2c)(b+2c) & 2b(4b^4+27b^3c+64b^2c^2+56bc^3+16c^4 \end{bmatrix}\begin{bmatrix} x \\ x^* \end{bmatrix}$$

$$= a(b+2c)\begin{bmatrix} 1 \\ (3b+2c)(b^2+6bc+4c^2) \end{bmatrix}$$

Solving the linear equaltions yields

$$x = \frac{2(b+c)^2 a}{b(b+2c)(4b+3c)} = \frac{2(1+t)^2}{(1+2t)(4+3t)} \cdot \frac{a}{b},$$

$$x^* = \frac{(3b+2c)a}{2b(4b+3c)} = \frac{(3+2t)}{2(4+3t)} \cdot \frac{a}{b}.$$

After substitution we get:

$$q = \frac{2(1+t)^2}{(1+2t)(4+3t)} \cdot \frac{a}{b},$$

$$q^* = \frac{1+4t+2t^2}{(1+2t)(4+3t)} \cdot \frac{a}{b};$$

and

$$\Pi = \frac{2(1+t)^3}{(1+2t)(4+3t)^2} \cdot \frac{a^2}{b};$$

$$\Pi^* = \frac{(4+27t+64t^2+56t^3+16t^4)bx^{*2}}{(3+2t)^2(1+2t)}. \qquad \textbf{(3)}$$

Note that when there is perfect flexibility (i.e. c = 0), $q = a/2b$ and $q^* = a/4b$, which is the standard outcome of a Stackelberg-leadership model.

Subsidy Policy

Now consider the case in which the Home government offers an export (i.e. production) subsidy to its national monopolist. With a specific subsidy, denoted *s*, the Home firm's problem is now given by:

$$\max_q \Pi = q(a - bq - bq^* + s) - c(q - x)^2.$$

The first-order conditions for equilbrium outputs are:

$$a - 2bq - bq^* + s - 2c(q - x) = 0,$$
$$a - 2bq^* - bq - 2c(q^* - x^*) = 0,$$

from which we get

$$\bar{q} = \frac{2(b+c)(a+2cx+s) - b(a+2cx^*)}{(3b+2c)},$$

$$\bar{q}^* = \frac{2(b+c)(a+2cx^*) - b(a+2cx+s)}{(3b+2c)}.$$

The Home government seeks to maximize the net of subsidy profit, i.e.

$$\max_s W = \bar{q}(a - b\bar{q} - b\bar{q}^*) - c(\bar{q} - x)^2 = (b + c)\bar{q}^2 - cx^2 - \bar{q}s,$$

which has the first-order condition

$$\frac{\partial W}{\partial s} = 2(b + c)\bar{q}\frac{2(b+c)}{(3b+2c)(b+2c)} - \bar{q} - s\frac{2(b+c)}{(3b+2c)(b+2c)} = 0.$$

Rearranging this, we have that

$$b^2\bar{q} = 2(b+c)s.$$

Substituting this into the Home firm's output rule, we get

$$\bar{s} = \frac{2b^2(b+c)(a+2cx) - b^3(a+2cx^*)}{4(b+c)(b^2+4bc+2c^2)}.$$

Using this result, we find

$$\hat{q} = \frac{2(b+c)(a+2cx) - b(a+2cx^*)}{2(b^2+4bc+2c^2)},$$

$$\hat{q}^* = \frac{(3b+2c)(b+2c)(a+2cx) - 2b(b+c)(a+2cx^*)}{4(b+c)(b^2+4bc+2c^2)}.$$

Note that these are the same solution values that we obtained for the quantity control case, implying that for given plant sizes both quantity control and subsidy policies produce identical outputs from the firms.

The subsidy and quantity control cases do, however, differ when we explicitly incorportate the plant size decision, since the Home firm will choose its plant size to maximize profits subject to the subsidy. Recall that the firm's problem is

$$\max_x \Pi = q(a - b\hat{q} - b\hat{q}^* + \bar{s}) - c(\hat{q} - x)^2.$$

The first-order conditions for the two firms are

$$\frac{\partial \Pi}{\partial x} = 2(b+c)\hat{q}\frac{4c(b+c)}{2(b^2+4bc+2c^2)} - 2cx = \frac{4c(b+c)^2}{b^2+4bc+2c^2}\hat{q} - 2cx = 0,$$

$$\frac{\partial \Pi^*}{\partial x^*} = 2(b+c)\hat{q}^*\frac{2c(3b+2c)(b+2c)}{4(b+c)(b^2+4bc+2c^2)} - 2cx^* = \frac{(3b+2c)(b+2c)}{b^2+4bc+2c^2}\hat{q}^* - 2cx^* = 0,$$

from which we get

$$\hat{q} = \frac{b^2+4bc+2c^2}{2(b+c)^2}x, \qquad \hat{q}^* = \frac{2(b^2+4bc+2c^2)}{(3b+2)(b+2c)}x^*. \tag{4}$$

Note that, unlike the quantity control case, the Home firm chooses a larger than optimal plant size in the sense that it produces at less than MES in equilbrium.

To get the full solution values, we use equations (4) to get the system

$$\begin{bmatrix} b(b^3 + 4b^2c + 8bc^2 + 4c^3) & 2bc(b+c)^2 \\ 4bc(b+c)(3b+2c)(b+2c) & 2b(4b^4 + 27b^3c + 64b^2c^2 + 56bc^3 + 16c^4) \end{bmatrix} \begin{bmatrix} x \\ x^* \end{bmatrix} = a(b+2c) \begin{bmatrix} (b+c)^2 \\ (3b+2c)(b^2 + 6bc + 4c^2) \end{bmatrix}$$

This can be solved to give

$$x = \frac{4(1+t)^4(1+2t)}{4 + 27t + 76t^2 + 118t^3 + 88t^4 + 24t^5} \cdot \frac{a}{b},$$

$$x^* = \frac{(3+2t)(1+2t)(1+2t+6t^2+4t^3)}{2(4 + 27t + 76t^2 + 118t^3 + 88t^4 + 24t^5)} \cdot \frac{a}{b}.$$

and

$$\Pi = \frac{(1 + 4t + 8t^2 + 4t^3)bx^2}{4(1+t)^3},$$

$$\Pi^* = \frac{(1 + 27t + 64t^2 + 56t^3 + 16t^4)bx^{*2}}{(3+2t)^2(1+2t)^2}, \qquad \textbf{(5)}$$

$$W = \frac{(1 + 2t + 10t^2 + 12t^3 + 4t^4)bx^2}{8(1+t)^5}.$$

These are all functions of the degree of production flexibility (captured by t). We will discuss the effect of t on equilibrium capacity, output and welfare in the final section. However, it is worth noting here that, at maximum flexibility (i.e. the Brander-Spencer case where $c = 0 \Rightarrow t = 0$), $q = 1/2$, $q^* =$

1/4, $= a^2/8b$, and $* = a^2/16b$. These are the same values as derived from the quantity control case where c = 0.

Industrial Policy with *ex post* Production Flexibility

As the introduction has already sugested, this paper identifies industrial policy with policies that seek to affect productive structure, specifically the size of the firm. The industrial policy game has the Home government offering a subsidy to productive capacity in stage 1, both national monopolists choosing capacity in stage 2, and output in stage 3. Since the quantity decision stage is the same as in the non-intervention case, we focus on fhe fims' plant size decision. With a subsidy to capacity of *h*, the Home firm's profit function is now

$$\Pi = (b+c)\bar{q}^2 - cx^2 + hx.$$

The first-order conditions for the capacity choice problem are now

$$\frac{\partial \Pi}{\partial x} = 2(b+c)\bar{q}\,\frac{4c(b+c)}{(3b+2c)(b+2c)} - 2cx + h = 0,$$

$$\frac{\partial \Pi^*}{\partial x^*} = 2(b+c)\bar{q}^*\,\frac{4c(b+c)}{(3b+2c)(b+2c)} - 2cx^* = 0,$$

from which we get

$$\hat{q} = \frac{(3b+2c)(b+2c)(2cx-h)}{8c(b+c)^2},$$

$$\hat{q}^* = \frac{(3b+2c)(b+2c)}{4c(b+c)^2}x^*.$$

Using these we construct the system

$$\begin{bmatrix} 2bc(9b^3+32b^2c+40bc^2+16c^3) & 16bc^2(b+c)^2 \\ 8bc(b+c)^2 & b(9b^3+32b^2c+40bc^2+16c^3) \end{bmatrix} \begin{bmatrix} x \\ x^* \end{bmatrix}$$

$$= \begin{bmatrix} 8c(b+c)^2(b+2c)a + h(3b+2c)^2(b+2c)^2 \\ 4(b+c)^2(b+2c)a \end{bmatrix}.$$

Solving this system, and denoting the equilibrium values under non-intervention with "ni" (given in equations 1), yields

$$\bar{x} = x_{ni} + \frac{(3b+2c)(b+2c)(9b^3+32b^2c+40bc^2+16c^3)}{2bc(3b^2+6bc+4c^2)(9b^2+22bc+12c^2)}h,$$

$$\bar{x}^* = x_{ni} - \frac{8c(b+c)^2(3b+2c)(b+2c)}{2bc(3b^2+6bc+4c^2)(9b^2+22bc+12c^2)}h.$$

The Home government chooses the subsidy to maximize national welfare defined as Home firm profits net of the subsidy:

$$W = (b+c)\hat{q}^2 - c\bar{x}^2 = \frac{(3b+2c)^2(b+2c)^2}{64c^2(b+c)^3}(h-2c\bar{x})^2 - c\bar{x}^2.$$

The first-order condition for this problem is

$$\frac{\partial W}{\partial h} = \frac{(3b+2c)^2(b+2c)^2}{64c^2(b+c)^3}2(h-2c\bar{x})\left(1 - 2c\frac{\partial \bar{x}}{\partial h}\right) - 2c\bar{x}\frac{\partial \bar{x}}{\partial h} = 0.$$

This can be solved for h to give

$$h = \frac{32t^2(1+t)^3(3+6t+4t^2)}{(3+2t)(1+2t)(81+432t+928t^2+1008t^3+560t^4+128t^5)}a.$$

The next section dicusses these results in more detail, but it is worth noting here that in the Brander-Spencer case ($c = 0$) $h = 0$. That is, industrial policy is completely ineffective so the output is the same as that in the laissez-faire case.

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