

Threshold uncertainty
in the
private-information subscription game

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3. A pervasive non-uniqueness of equilibria hinders comparative statics analysis and muddles efficiency considerations.

literature on lobbying surveyed in Potters and Sloof (1996) and to the papers analyzing advantages and disadvantages of heterogeneity and fragmentation in cooperative endeavors, a literature surveyed in Alesina and La Ferrara (2005). Moreover, our results complement McBride's (2006). While McBride (2006) focuses on the effects of changing the distribution of the threshold cost, we focus on the effects of changing the distributions of players' values. One of McBride's most interesting results is that increased uncertainty in the distribution of the threshold may increase equilibrium contributions,² thus contradicting the usual in-

where r

where $d(s_i; s_i) = \sup_{v_i} |s_i(v_i) - s_i(v_i)|$: We prove (8) for O_1 only, because the proof for O_2 is identical, and for brevity, we indicate $d(s_1; s_1)$ with d

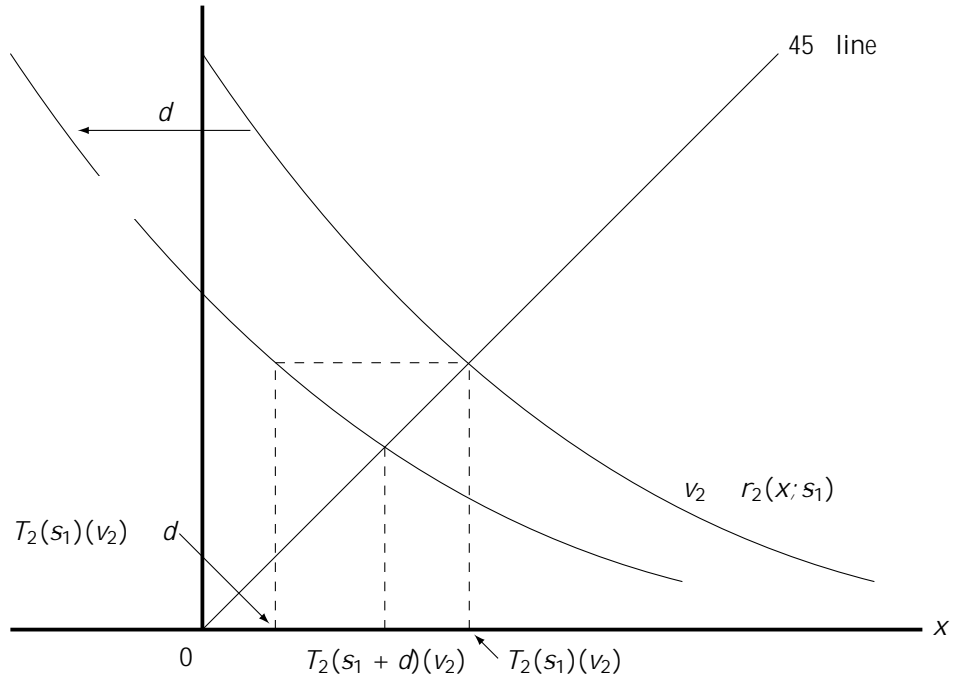


Figure 2: Intermediate step in establishing that the operator O_i exhibits "discounting"

Observe from (4) that $r_2(x; s_1 + d) = r_2(x + d; s_1)$, so that the curve labelled $v_2 \quad r_2(x; s_1 + d)$ is simply a leftward translation of the original curve, label that

this contradiction shows it b1nustws

Ex post efficiency would require that the public good be provided if and only if $v_1 + v_2 \geq c$. If, further, c is uniformly distributed over $[0; 2]$ and all random variables are independent, then the good would be efficiently provided if and only if $v_1 + v_2 \geq c$.

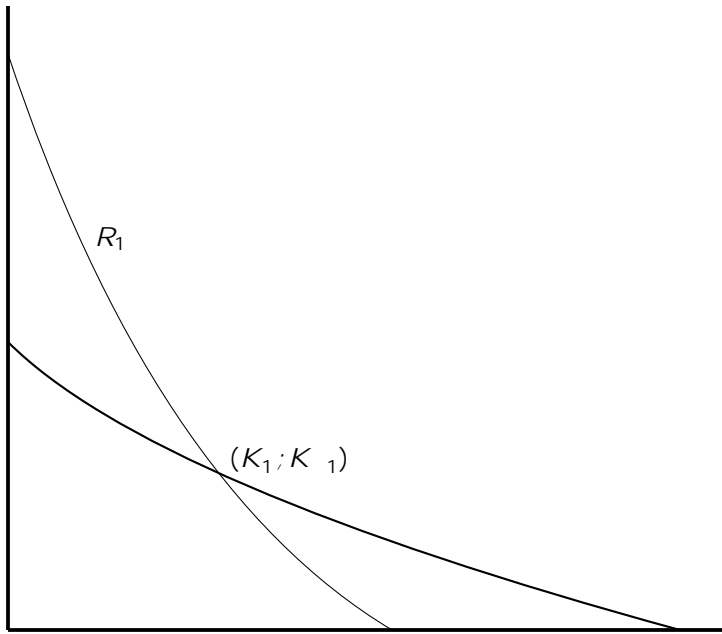
The threshold value

are reduced in the sense of first-order stochastic dominance. We see that player 1's expected contribution falls and player 2's increases. Indeed, the change from F_1 to \hat{F}_1 results in a weak reduction in player 1's contribution strategy and a weak increase in player 2's.

3.1 Crowding-out

We denote with K^E the level of contributions that are exogenously provided by an external authority; and we consider how the players' contributions change as K^E increases. It is immediate to replicate the steps leading to Proposition 3 and obtain that in equilibrium, the system of equations (15) takes the form

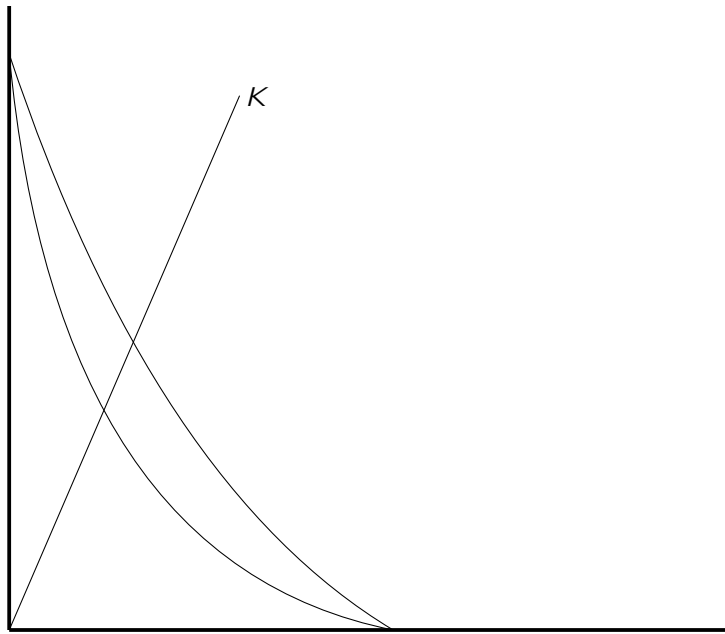
$$g_i; 0 = K_i^{-1}$$



that if F_1 first-order stochastically dominates \hat{F}_1 , then the expected value of any increasing function of v_1 is at least as large under F_1 as under

contribute less, in aggregate. To see this last point, observe that, for $j \notin 1$,

$$\prod_{i \notin j} \hat{K}_i = \prod_{i \notin j} \hat{K}_i \hat{K}_j \prod_{i \notin j} K_i \quad K_j = \prod_{i \notin j} K_i;$$



4.1 General Considerations

The notion of efficiency we use is *interim incentive efficiency*, defined by Holmstrom and Myerson (1983), and further characterized by Ledyard and Palfrey (1999 and 2007). This is the same notion used by Laussel

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Proposition 6 (A sufficient condition for inefficiency). *Let $(p; x)$ be an incentive compatible, individually*

in Ledyard and Palfrey (2007)) imply each type of the buyer is not hurt by the change in mechanisms, and each type of the seller is strictly better off.¹³ This establishes that p is Pareto-dominated by \hat{p} . \square

The subscription game is an incentive compatible, individually rational (non-direct) mechanism; for any equilibrium (s_1, \dots, s_n) , using the optimal behavior of the collector described above, the equilibrium allocation rule is the following:

$$p^{eq}(v; c) = \begin{cases} 1 & \text{if } \sum_i^P s_i(v_i) \geq c \\ 0 & \text{if } \sum_i^P s_i(v_i) < c \end{cases}$$

5 Conclusion

This paper has picked up where Nitzan and Romano (1990) left off, calling for their model with threshold

be provided (see Alboth *et al.*, 2001, and Barbieri and Malueg, 2008), which introduces some multiplicity of equilibria. A second area for research would fully investigate how differences among individuals or groups might hinder or facilitate interactions. Our framework offers a laboratory for studying such effects where interactions with less familiar groups could be modeled through the relative dispersion in perceived values of members of own versus different groups.

Appendix

Proof of Proposition 1.

that $T_2(s_1)(v_2)$ is well-defined by the assumption that s_1 is part of an equilibrium. We can now show $T_2(s_1 + d)(v_2) = T_2(s_1)(v_2)$. Clearly, the statement is true if $T_2(s_1 + d)(v_2) = 0$; so by way of contradiction assume $T_2(s_1 + d)(v_2)$

statement is true if $T_2(s_1)(v_2) = 0$;

as we find one such value, and we keep increasing K^N , non-contributors remain non-contributors and P_i

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