

Share Equilibrium in Local Public Good Economies

by

A. van den Nouweland

Abstract: We defi

these.

Recall that a (pure) public good is a commodity that can be consumed

taken approaches in a similar spirit, with individuals paying shares of costs rather than per unit prices, include Kaneko (1977a,b) and Mas-Colell and Silvestre (1989,1991).

In the current paper, we study local public good economies (or, in other words, economies with clubs). The defining features of a local public good economy are that the public good can be consumed in its entirety by members of a jurisdiction while non-members are excluded from consumption. A share equilibrium for an economy with local public goods, a share index of a player determines his share of costs in any jurisdiction he might join. Roughly, a share equilibrium includes a specification of a partition of the

to live in specific neighborhoods based, at least in part, on the quality of the public school (i.e., the level of local public good provision) and the property

vide an axiomatic characterization of the share equilibrium.⁵

condominium home-owners association, some subset of owners may choose not to attend, knowing that they will still have to pay their shares of the costs of the association. These owners become non-decision makers, taking their shares of the condominium fees as given and relinquishing the power to decide on all matters affecting the association to the attendees.

Formally, a local public good economy is a list

To illustrate the cost function and its interpretation, let b be the set of condominium owners in a building and let X be the set of owners who attend a particular meeting of the condominium home-owners association. In our terminology, the owners in $b \setminus X$ become non-decision makers. However, not attending the meeting does not absolve the absent owners from paying their

4 Share equilibrium and the core

In this section we explore relations between share equilibria and the core of a local public good economy.

We distinguish between $z_D(\pi Ch) = 0$ and $z_D(\pi Ch) \neq 0$.

Suppose $z_D(\pi Ch) \neq 0$.

5 Existence of a share equilibrium

We have defined share equilibrium because we are interested in extending the ratioequilibrium concept to a local public good. Now that we have identified a new equilibrium concept, there are, of course, many questions that arise. One of these questions concerns existence. While we defer an extensive study of the existence of the share equilibrium to work, we identify a set of sufficient conditions for existence of the share equilibrium in this section.

Players in a local public good economy have preferences over private and local public good consumption and over the jurisdiction members with which they share the local public good. A local public good economy is symmetric if all its players play the same role and have the same preferences and initial endowments. Note that, because players enter into one another's utility functions, this necessarily implies that players care only about how many players are in their jurisdiction and not about their identities. Technically, a local public good economy (Y, h, X)

(i) $(x_i)_{i \in N}$; Z_D is symmetric if E is symmetric.

players in $b \setminus f$

of decision makers $f \in Q^{11}$ and let $(s(x, y, P)) \in \mathbf{R}^{D(E)} \times W(b(Y))$. The reduced economy of Y with respect to f and $(s(x, y, P))$ is the economy in which the set of decision-making players is f , so

$$Y^{R, (s, (x, y, P))} = \{ (i, (c_i)_{i \in N}; (k_i)_{i \in N}, \omega_i) \}$$

and with the utility function

$$z_i(\pi^i) = \prod_{j \in R} \pi_j^{a_{ij}} z_{D_j}(\pi^i)$$

for all $\pi \in \mathbf{R}_+$ and $a_{ij} \in [0, 1]$.

The idea behind the defi

not influence the final outcome of the process. The consistency property is defined using reduced economies. A solution σ on E is consistent (CONS) if it satisfies the following condition.

If $Y \in E$, $(\sigma(Y), P)$ is a reduced economy

for all $\epsilon \in \mathbf{R}_+$

one decision-making player $Y = \{(\alpha_j)_{j \in N}; (\beta_j)_{j \in N}; z_{\{i\}}\} \in F$ it holds that

$$Z(Y) = \{(\alpha_j)_{j \in N} \mid \alpha_j \geq 0, z_{\{i\}} = \sum_{j \in N} \alpha_j \beta_j\} +$$

Theorem 3 shows that consistency, converse consistency, and one-person rationality characterize the share equilibrium.

Theorem 3. The share equilibrium is the unique solution on E that satisfies one-person rationality, consistency, and converse consistency.

Proof. In Lemmas 1 and 2 we proved that the share equilibrium satisfies CONS and COCONS. To show that the share equilibrium satisfies OPR, let $Y = \text{hb } Q \{ \}; (' j j N \quad j j N$

sizes," *Journal of Economic Theory*, doi:10.1016/j.jet.2007.07.06.

