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Channel network scaling laws in submarine basins

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[1] Fluvial drainage basin area is often related to channel length and local slope through power law relationships and the relatively small range of exponents observed in these relationships is thought to result from physical mechanisms. Proposed mechanisms assume that the observed correlation between drainage area and fluid discharge is caused by precipitation. Using high resolution DEMs of channelized continental slope settings offshore Monterey, CA and Brunei Darussalam we extracted submarine channel profiles and drainage area statistics from five basins. In-situ and remote observations suggest discharge in these oceanic settings is determined by boundary conditions at the shelf-edge. In spite of substantial differences in environment and physical process, the data yield submarine scaling exponents within the range of terrestrial (fluvial) observations. The convergence in scaling relationships from two very different settings supports theoretical arguments that channel network structure results from the aggregation of random walks.
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1. Introduction

[2] Technological advances over the last 25 years in acoustical sonar systems now allow the collection of bathymetric maps with spatial resolution comparable to terrestrial digital elevation models (DEMs) [*Fildani and Normark*, 2004; *Garcia et al.*, 2005]. Numerous bathymetric surveys of continental shelf and slope settings reveal dendritic channel networks [*Garcia et al.*, 2005; *Puig et al.*, 2003] that are qualitatively similar to their terrestrial cousins. The primary mechanisms that initiate and drive the evolution of these channel networks are sediment gravity flows [*Heezen and Ewing*, 1952], namely turbidity currents. It is hypothesized that in some settings these flows are responsible for setting the gradient of delta fronts [*Kostic et al.*, 2002]. Unfortunately, the great water depths at which many of these systems exist and the infrequency of submarine channelized flow events have limited the quantity of direct dynamic observations [*Xu et al.*, 2004]. To overcome this problem, researchers have attempted to infer dynamic processes from static morphology and stratigraphy [*Mitchell*, 2005; *Pirmez and Imran*, 2003].

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[3] Studies of terrestrial channels have related the structure of drainage basins to specific physical processes, such as fluvial bedrock incision [*Rigon et al.*, 1996; *Willgoose et al.*, 1991]. These physical mechanisms are rooted in field [e.g., *Snyder et al.*, 2000] observations, and tested against landscape evolutions models [e.g., *Tucker and Whipple*, 2002]. Two power-law relationships that quantify the structures of terrestrial drainage basins have garnered great attention: Hack's law relating channel length (l) to basin area (a); and the relationship between channel slope (S) and basin area.

[4] The scaling form of Hack's law and slope-area relationships is frequently attributed to correlations between contributing area and fluid discharge [*Rodriguez-Iturbe et al.*, 1992; *Snyder et al.*, 2000]. The physical explanation for this correlation lies in the method of fluid input to terrestrial channel networks. If precipitation is independent of channel network pattern, it follows that rainfall capture increases more or less linearly with drainage area. Overland and shallow subsurface flow follows local paths of steepest descent. As a result, contributing upslope area at any point in a channel network is a proxy for net fluid discharge. In submarine settings it seems obvious that contributing area is not related to fluid discharge in a manner quantitatively analogous to terrestrial environments (Figure 1). Discharge is primarily determined by network boundary conditions at the up slope perimeter of the drainage basin. Unlike terrestrial channel networks, sediment input to submarine networks can be localized to canyon heads. Downslope of the shelf edge the discharge of a turbidity current evolves due to deposition or erosion of sediment – a result of local fluid dynamics – rather than increasing rainfall runoff. Other differences exist between transport processes in terrestrial and submarine environments. In the terrestrial water discharge drives flows downstream, while in the submarine sediment discharge is primarily responsible for driving flow downstream. One might therefore expect that different boundary conditions and transport processes in submarine settings would result in scaling exponents that differ from terrestrial values.

2. Study Regions

[5] A high resolution DEM of the continental shelf offshore Monterey, CA was constructed from bathymetric data collected by a multibeam sonar system in 1998. The survey encompassed 16780 km^2 and four submarine canyon systems. The DEM has evenly spaced horizontal bins of 40 m. Imaged canyon systems from north to south were Ascension (M1), Monterey (M2), Sur (M3), and Lucia (M4) (Figure 2a). Two-dimensional (2D) seismic surveys oriented perpendicular to the canyon axis indicate that these canyons have formed by erosion into the continental shelf [*Fildani and Normark*, 2004]. The continental shelf offshore Monterey

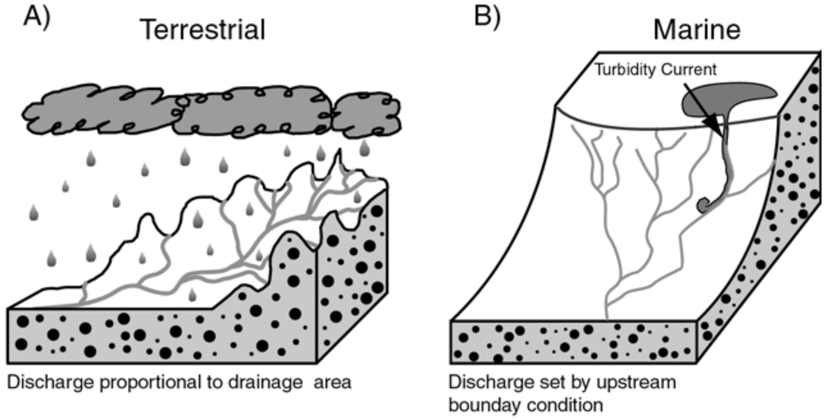

Figure 1. Schematic diagrams illustrating difference in terrestrial and submarine flow boundary conditions. (a) In terrestrial drainage basins discharge along channel profiles is related to contributing basin area as a result of precipitation. (b) In many submarine environments, flow discharge is set by boundary condition at continental shelf-edge which delivers sediment to the slope environment.

is extremely narrow. Canyon heads initiate less than a kilometer from the shoreline. In-situ measurements have linked the timing of gravity flows in this region to local storm and flood events [*Xu et al.*, 2004]. The network structure of channels offshore Monterey, CA was studied by *Pratson and Ryan* [1996]. They found that Horton statistics describing ordering of channels offshore Monterey, CA were similar to statistics describing terrestrial systems and suggested an acausal explanation for the similarity.

[6] An industry-grade 3D seismic survey covering 555 km^2 was used to create a DEM of the continental slope offshore Brunei Darussalam, which encompasses a tributary network of submarine channels (B1) (Figure 2b). The DEM has evenly spaced horizontal bins of 25 m. The width of the continental shelf in this region varies between 50 and 70 km. All channels in this network initiate approximately 2 km downslope of the shelf-edge (~300 m water depth). Widths and depths of channels rapidly increase over the first 5 km of downslope

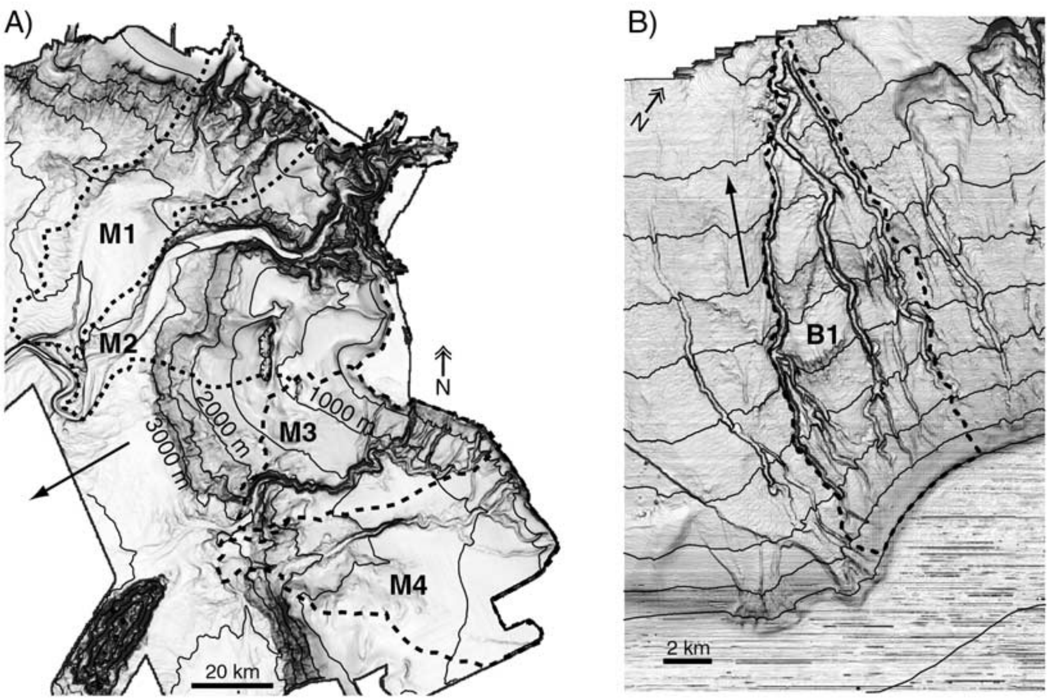

Figure 2. Mean surface gradient maps of study regions. (a) Map of continental slope offshore Monterey Bay with 500 m contours of bathymetry. Margins of drainage basins M1–M4 are defined by black dashed lines. Arrow indicates general flow direction. (b) Map of continental slope offshore Brunei Darussalam with 100 m contours of bathymetry. Margins of drainage basin B1 is defined by black dashed lines. Arrow indicates general flow direction.

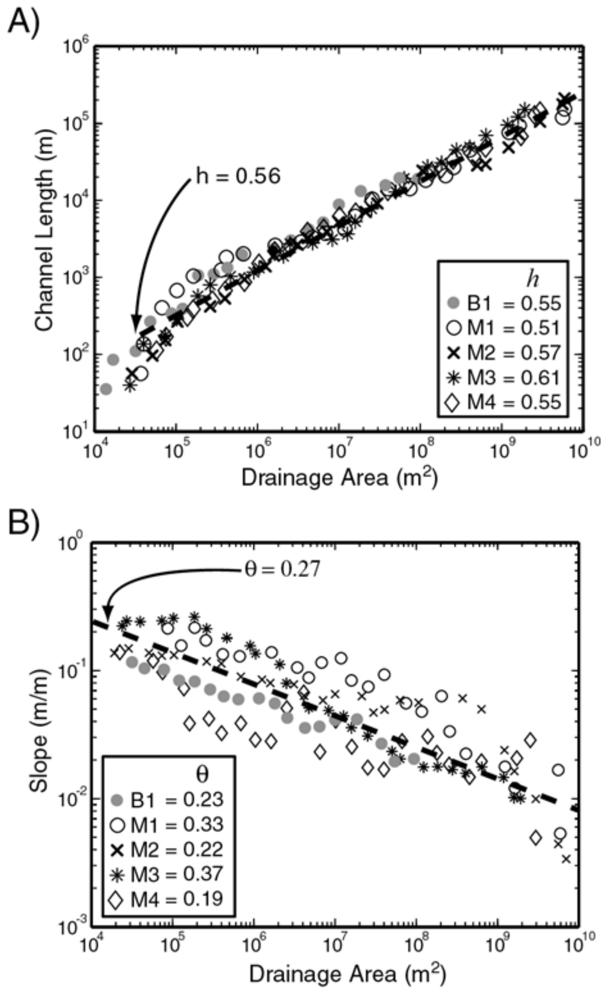

Figure 3. Observed scaling relationships for 5 drainage basins. Regressions were performed between drainage areas of 104–1010. (a) Relations for channel length to contributing basin area in study regions. The best-fit values for *h* from each basin are shown in bottom right of graph. Black dashed line is best-fit through all data. (b) Relations for channel gradient to contributing basin area in study regions. The best-fit values for θ from each basin are shown in bottom right of graph. Black dashed line is best-fit through all data.

distance and then remain approximately constant for the next 13 km, the extent of the region imaged. Subsurface mapping of this region indicates that the channel network evolved by deposition (rather than erosion) onto the continental shelf. Channel head locations and downslope trends in channel width and depth suggest that channel forming flows were initiated by sediment evacuation events on the continental shelf, possibly due to storm activity.

3. Hack's Law

[7] A scaling relationship between contributing area, a, and the length of a basin's main stream, l:

$$l \propto a^h \tag{1}$$

was first observed by *Hack* [1957] in a study of rivers in Virginia and Maryland. Subsequent studies found similar relationships for channels in other geographical regions and have empirically delineated a range for Hack's exponent, h, between 0.5 and 0.7 [*Dodds and Rothman*, 2000]. To date, a consensus surrounding the cause(s) of the observed range of h has not been achieved. Proposed explanations range from minimization of energy associated with channelized fluid flow [*Rodriguez-Iturbe et al.*, 1992] to the inevitable consequence of random walks [*Dodds and Rothman*, 2000].

[8] We extract channel profiles and drainage basin statistics from DEMs of our two study regions in a manner analogous to terrestrial drainage basin analysis [*Snyder et al.*, 2000]. In the terrestrial environment, the upslope boundary of a drainage basin is associated with topographic highs that separate the path of fluid flow between neighboring basins. This definition does not hold for submarine drainage basins as a result of linked terrestrial environments upslope of marine basins. For this study we assume that submarine flow events initiate at the continental shelf-edge (defined as the 100 m contour offshore Monterey, the 200 m contour offshore Brunei Darussalam) and use this to define the upslope extent of drainage basins.

[9] Channel length vs. contributing area trends were calculated for basins M1–M4 and B1. Data was binned at intervals of $\log_{10}a = 0.2$ to ensure least-squares regression trends were not biased to channel lengths at high contributing areas (Figure 3a). Hack's exponent, h, ranged from 0.51–0.61 with an average of 0.56 for our 5 submarine drainage basins. Data demonstrate a convincing power-law fit spanning 6 orders of drainage area magnitude, with $R^2 > 0.95$ for all basins. Surprisingly, the channel length intercept of the regression line through the Brunei Darussalam basin fell within the range of channel length intercepts measured for the Monterey channels.

4. Concavity

[10] Statistical measurements defining network structure have been used to assess regional tectonic [*Snyder et al.*, 2000] histories in erosional environments and to validate numerical landscape evolution models [*Tucker and Whipple*, 2002]. A commonly reported statistic of drainage basins is an exponent termed concavity (θ) that relates the contributing area to channel gradient:

$$S = k_s \, a^{-\theta}, \tag{2}$$

where the coefficient k_s is termed channel steepness. Studies of river networks over a range of geological and environmental conditions report an empirical concavity range between 0.1–0.7 [*Whipple and Tucker*, 1999]. Theoretical studies have related concavity values to mechanistic erosion models [*Whipple and Tucker*, 1999]. Alternatively, some studies have assigned a regionally-averaged (fixed) concavity value to investigate local changes in channel steepness among drainage basins, which has been linked to rates of tectonic uplift.

[11] Channel elevation profiles were extracted from the 5 study drainage basins in a manner analogous to the extraction of basin area and channel length. Downslope gradient was measured along channels using linear regression in a moving window of 160 m for Monterey profiles and

Table 1. Comparison of Hack's Law for Theoretical and Real Channel Networks

Network	h
Non-convergent flow	1
Directed random	2/3
Undirected random	5/8
Optimal Channel Networks	1/2–2/3
Real Rivers	0.5–0.7
Submarine channels (this study)	0.51–0.61

150 m for Brunei profiles. As with channel length, data were binned at intervals of $\log_{10}a = 0.2$ (Figure 3b). Data was fit using normal least squares regression, which is justified by the high ratio of error in channel slope compared to contributing area [*Flint*, 1974]. Concavity ranged from 0.19 to 0.37 with an average of 0.27 for our 5 submarine drainage basins. Data defining the slope-area trend span 6 orders of drainage area magnitude, with $R^2 > 0.71$ for all regressions.

5. Discussion

[12] A study of canyon long-profiles and network scaling laws on the Atlantic continental slope found a similar range of Hack's exponent and concavity to our measurements [*Mitchell*, 2005]. Measured h and θ in both studies are well within the range of observed terrestrial values. *Mitchell* [2005] hypothesized that submarine concavity values were similar in magnitude to terrestrial values because of the cumulative downslope effect of landslide-triggered turbidity currents. In this model, the cumulative time a channel acts as an active flow conduit increases down axis due to a uniform distribution of landslides within a drainage basin. This model assumes the primary mechanism delivering sediment to the slope is hemipelagic fallout. Direct flow observations and seismic geomorphology studies offshore Monterey, CA and Brunei Darussalam do not support a landslide-driven connection between basin area and flow frequency. Our measurements of h and θ are therefore surprising given the difference in fluid discharge boundary conditions and physical transport processes in the two environments.

[13] Convergence of channel network scaling relationships in two very different environments poses an interesting question. Are unknown processes conspiring in the submarine environment to produce similar h and θ exponents through entirely different means, or are these relationships simply the result of random processes and/or insensitive to landscape evolution processes? We explore the latter option through comparison of our observations with two theoretical and numerical studies of acausal (random) relations between area and drainage basin structure [*Dodds and Rothman*, 2000; *Schorghofer and Rothman*, 2002].

[14] Scaling and similarity related to the planform structure of river networks were reviewed by *Dodds and Rothman* [2000]. This study examined Hack's law, Horton's law of stream numbers, and Langbeins's law relating the total length of all streams in a network to basin area. To assess and compare various channel network evolution models, Hack's exponent was calculated for each model output. Models assessed included non-convergent flow, directed random networks, undirected random networks, and optimal channel networks. A comparison of Hack's exponent in these four models, real rivers, and our submarine networks is shown in Table 1. *Dodds and Rothman* [2000] concluded that ascribing geological meaning to measured values of h is risky due to the overlapping range of h in real rivers and random networks. Observed h values in our submarine networks are slightly less than the mean h values predicted by random walk networks but cluster within the range of h values resulting from random network models. Similar scaling exponents in terrestrial and submarine environments support channel planform scaling relationships derived from random walk models.

[15] An acausal link between drainage area and slope for channels with concavity up to 1/3 was predicted by *Schorghofer and Rothman* [2002] and confirmed via network analysis of surfaces unrelated to channelized fluid flow. For streams that aggregate due to convergent topography, even where rivers do not carve their own paths, a systematic inverse correlation can be expected between slope and area. This relationship is a result of an inverse relationship between contour curvature and local surface slope and a positive correlation between basin area and curvature. [*Schorghofer and Rothman*, 2002].

[16] *Schorghofer and Rothman* [2002] provide a quantitative test to distinguish surfaces where concavity can be used to assign geological meaning and surfaces where a slope-area relationships results from the geometric relationship (3). This is done using the Hurst exponent H, which quantifies the dependence of elevation fluctuations (roughness) on the length scale of measurement. The Hurst exponent can be determined from the power law relationship:

$$\sigma(w) \sim L^H \tag{3}$$

where L is the length of a measurement window and $\sigma(w)$ is the structure function of topography over a length L:

$$\sigma = \left[\frac{1}{L}\sum_{i=1}^{L}\left(h_{i+j} - h_i\right)^2\right]^{1/2} \tag{4}$$

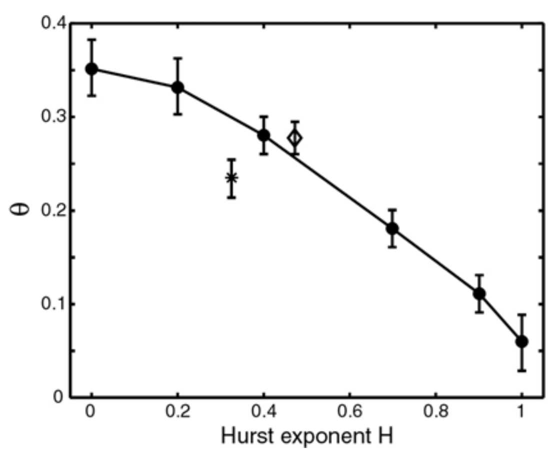

Figure 4. Relationship between concavity and Hurst exponent for Gaussian surfaces over the range of Hurst exponents of natural landscapes [*Schorghofer and Rothman*, 2002]. The diamond and star with error bars correspond to the Monterey basins and the Brunei Darussalam basin respectively.

and h_i is the difference between the height at a location i and the mean height in the measurement window. For random Gaussian topography there is a characteristic relationship between concavity and Hurst exponent [*Schorghofer and Rothman*, 2002], which we compare to our measured values in Figure 4. A surface is called Gaussian if the phases of its Fourier modes are random and uniformly distributed. Gaussian surfaces can be as smooth as natural channel topography [*Adler*, 1981]. We find that our values of θ and H cannot be distinguished from random surfaces and therefore suggest the slope-area relationships observed in our study regions cannot be used to assign geological significance.

6. Summary

[17] Empirical relationships between contributing basin area and channel network structure first measured in terrestrial environments have been quantified for 5 submarine drainage basins. The terrestrial range of observed values of Hack's exponent and concavity encompass observations on submarine networks in this study. Discharge boundary conditions in these submarine networks are linked to sediment transport conditions at the shelf-slope break. As a result, drainage basin area is not related to flow discharge in a manner analogous to terrestrial systems. Measurements of basin structure support the acausal relations for Hack's Law reviewed by *Dodds and Rothman* [2000] and the acausal slope-area relation predicted by *Schorghofer and Rothman* [2002]. Further, our study suggests caution in using only channel network scaling relations for interpretation of submarine or extraterrestrial environments. Though the structure of channel networks might be random, additional channel topographic characteristics may contain useful data for interpretation of submarine transport processes. These measures include, but are not limited to, relationships between channel width, depth, and velocity which systematically vary as a function of transport regime in the terrestrial [*Church*, 2006].

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