2) Recreational drinking often results in drunk driving. You wish to model alcohol consumption in order to predict drunkenness from drinking behavior.

The arrows show the direction of alcohol movement.

Let $X$ be the input drinking rate.
$A_1 =$ amount of alcohol in body fluid.
$A_2 =$ amount of alcohol in body fat.

The liver consumes alcohol at a rate $\beta A_1$.

For a thin person with no fat $A_1 =$ input - output

$$A_1 + \beta A_1 = X \quad \text{(1)}$$

A fat person stores some alcohol in the body fat compartment. One may then say $A_1 =$ input + input from fat - output

$-\dot{A}_2 = $ input from fat $= \alpha_2 A_2 - \alpha_1 A_1$

Therefore

$$A_1 + \beta A_1 + \dot{A}_2 = X \quad \text{(2)}$$
$$\dot{A}_2 - \alpha_1 A_1 + \alpha_2 A_2 = 0 \quad \text{(3)}$$

Note that with no fat $\dot{A}_2 \equiv 0$ so eq'n (2) $\rightarrow$ eq'n (1).

Fat holds more alcohol than water (equal partial pressures). The constants $\alpha_1, \alpha_2, \beta$, are dependent upon solubility, volumes of the various compartments, and diffusion constants. $\alpha_1, \alpha_2, \beta$, are positive real numbers. Values are given in the supplement.
2 (Continued...)

a) A fat person and a thin person drink equal amounts at a constant rate $x_1$. After a long period which person is drunker (which has greater $A_i$)?

b) Both drink until they reach equilibrium blood alcohol levels of 0.2% ($A_i/V_i = 0.002$). They quit drinking simultaneously. How long will each remain legally drunk ($A_i/V_i \geq 0.001$)?

c) Both are sober. They each consume 5 drinks in a short period. Describe the outcome, including: who will get drunk? when? and for how long?

\[
\alpha_1 = 0.29 \text{ hr}^{-1} \\
\alpha_2 = 0.68 \text{ hr}^{-1} \\
\beta_1 = 0.25 \text{ hr}^{-1}
\]
1) hint: treat cases a & b as different transfer functions

2) $\alpha_1 = 0.24 \text{ hr}^{-1}$  
   $\alpha_2 = 0.68 \text{ hr}^{-1}$  
   $\beta_1 = 0.25 \text{ hr}^{-1}$

Assume: 1 drink contains 15 g alcohol

$V_i = 30 \text{ L}$ is a aqueas compartment available to alcohol

$\frac{A_i}{V}$ = alcohol concentration; for $A_i = 30$, then

$\frac{30 \text{ g}}{30,000 \text{ g}} = 0.001 = 0.1\% \Rightarrow$ legally drunk

$x = x(t) = \text{ input rate}$

Hints:  a) $x = R u(t)$ where $R$ is a constant, equal to (grams/hour) consumption
    
    b) your choice
    
    c) $x = (5 \text{ drinks})(15 \text{ g/drink}) \delta(t) = 75 \delta(t)$