EENS 634 The Earth

Homework IV: Isotope Geology

1.

a) Calculate the abundances of the isotopes and the atomic weight of strontium, Sr_o, which has the following isotopic ratios:

$^{87}Sr/^{86}Sr = 1.0$, $^{86}Sr/^{88}Sr = 0.1194$, and $^{84}Sr/^{88}Sr = 0.0068$

The masses of the various isotopes of strontium are:

$^{88}Sr = 87.9056$ amu, $^{87}Sr = 86.9088$ amu, $^{86}Sr = 85.9092$ amu, and $^{84}Sr = 83.9134$ amu.

b) Calculate the $^{87}Rb/^{86}Sr$ ratio (atomic) of a specimen of biotite that has the following concentrations:

Rb = 465 ppm, Sr = 30 ppm, and whose $^{87}Sr/^{86}Sr$ ratio is 1.000. Use the results of part (a) in this calculation.

Note that:

$^{87}Rb/^{86}Sr$ = (ppmRb/ppmSr) (at. wt. Sr_0/at. wt.^{87}Rb) (%^{87}Rb/%^{86}Sr)

where at wt.^{87}Rb = 85.46776 amu and %^{87}Rb = 27.8346.

c) If the initial $^{87}Sr/^{86}Sr$ of the biotite in part (b) was 0.7035, what is the age of this mineral, assuming $\lambda = 1.39 \times 10^{-11}\ y^{-1}$. Calculate model ages using both the approximate and exact equations.

Exact equation: $$\left(\frac{^{87}Sr}{^{86}Sr}\right)_t = \left(\frac{^{87}Sr}{^{86}Sr}\right)_0 + \left(\frac{^{87}Rb}{^{86}Sr}\right)_t \left(e^{\lambda t} - 1\right)$$

Approximate equation: $$\left(\frac{^{87}Sr}{^{86}Sr}\right)_t = \left(\frac{^{87}Sr}{^{86}Sr}\right)_0 + \left(\frac{^{87}Rb}{^{86}Sr}\right)_t \lambda t$$

d) What was the $^{87}Sr/^{86}Sr$ ratio of this biotite 250 million years ago, using the approximate equation? To answer this question, first figure out what you know. (i.e. you know $^{87}Sr/^{86}Sr$ and $^{87}Rb/^{86}Sr$ ratios today and the time since the process started, 250 million years. You don't know the $^{87}Sr/^{86}Sr$ ratio when the process started 250 million years ago nor do you know the $^{87}Rb/^{86}Sr$ ratio 250 million years ago, so you need to figure these out first.

2. Six samples of Sierra Nevada granodiorite have Sr and Rb isotopic ratios as follows:

SAMPLE	$^{87}Sr/^{86}Sr$	$^{87}Rb/^{86}Sr$
1	0.7117	3.65
2	0.7095	1.80
3	0.7092	1.48
4	0.7083	0.82
5	0.7083	0.66
6	0.7082	0.74

a) Find the equation of the isochron using least squares regression. (See note on linear regression on the last page or use the regression feature of Microsoft Excel).

b) What are the age and initial $^{87}Sr/^{86}Sr$ ratio of the intrusion?

c) Use this initial ratio to examine possible sources of the magma. It could conceivably (a) have come straight up from the mantle at the time of the intrusion, or (b) it could have been formed by melting of very old crustal rocks, or (c) it could have formed from the mantle sometime prior to intrusion, but not necessarily very long before, or (d) it could be a mixture of (a), (b), and (c).

To examine these possibilities, calculate the following using the basic equation below:

$$(^{87}Sr/^{86}Sr)_t = (^{87}Sr/^{86}Sr)_o + (^{87}Rb/^{86}Sr)_t\lambda t$$

i) $^{87}Sr/^{86}Sr$ in the mantle at the time of intrusion of the Sierra rock assuming an initial ratio in the mantle, 4.5 billion years ago, of 0.698 (as deduced from meteorites of that age) and a $^{87}Rb/^{86}Sr$ ratio in the mantle today of 0.1. Check that this $^{87}Rb/^{86}Sr$ is consistent with the present $^{87}Sr/^{86}Sr$ = 0.704 for the mantle, (as deduced from recent basalts). Again, remember that you do not know the $^{87}Rb/^{86}Sr$ ratio in the mantle at any time other than today.

ii) The $^{87}Sr/^{86}Sr$ ratio expected at the time of intrusion in crustal rocks separated from the mantle 3.7 billion years ago, assuming a $^{87}Rb/^{86}Sr$ ratio in crustal rocks today of 0.7.

iii) The time at which source material of the Sierra magma should have separated from the mantle to give the correct initial ratio at the time of intrusion, assuming the same present day crustal ratio of $^{87}Rb/^{86}Sr$ = 0.7, for this source material. Note that you will have to solve three equations in order to determine the time of separation of the crust from the mantle. You know the following: $^{87}Rb/^{86}Sr$ in the crust and mantle today, 0.7 and 0.1, respectively; the $^{87}Sr/^{86}Sr$ ratio required in the crust at the time of magma generation (the initial ratio calculated from the isochron equation); and the $^{87}Sr/^{86}Sr$ ratio of the mantle today. You don't know the $^{87}Sr/^{86}Sr$ in the crust today, the $^{87}Sr/^{86}$ ratio of the crust at the time it separated from the mantle, or the time before the present that the separation took place.

What do you conclude?

3. The decay of ^{238}U to ^{206}Pb can be described as

$$\left(\frac{^{206}Pb^*}{^{238}U}\right) = \frac{\left(\frac{^{206}Pb}{^{204}Pb}\right)_t - \left(\frac{^{206}Pb}{^{204}Pb}\right)_0}{\left(\frac{^{238}U}{^{204}Pb}\right)_t} = \left(e^{\lambda_{238}t} - 1\right)$$

where $\lambda_{238} = 1.551 \times 10^{-10}$/yr.

Similarly, the decay of ^{235}U to ^{207}Pb can be described as

$$\left(\frac{^{207}Pb^*}{^{235}U}\right) = \frac{\left(\frac{^{207}Pb}{^{204}Pb}\right)_t - \left(\frac{^{207}Pb}{^{204}Pb}\right)_0}{\left(\frac{^{235}U}{^{204}Pb}\right)_t} = \left(e^{\lambda_{235}t} - 1\right)$$

where $\lambda_{235} = 9.8485 \times 10^{-10}$/yr.

A uranium bearing mineral, like zircon, which satisfies all the assumptions of radioactive age dating will yield a concordant age from these two equations. The Concordia diagram is a plot of $\left(\frac{^{206}Pb^*}{^{238}U}\right)$ against $\left(\frac{^{207}Pb^*}{^{235}U}\right)$

Dates that are concordant plot on the Concordia. Construct the Concordia diagram from these two ratios for times between the present and 2.5 billion years before present, using the following time increments:

Interval (x 10^9) yrs. BP	Increment
0 - 0.1	0.01
0.1 - 2.0	0.1
2.0 - 2.5	0.5

4. Consider the following zircon data from several plutons in a batholith:

Sample	$\%^{204}Pb$	$\%^{206}Pb$	$\%^{207}Pb$	$^{238}U/^{204}Pb$	$^{235}U/^{204}Pb$
1	0.048	80.33	9.00	6819.315	49.487
2	0.048	79.68	8.97	8478.345	61.526
3	0.067	72.71	8.50	4799.873	34.832
4	0.050	80.51	8.91	8273.386	60.039
5	0.030	80.26	8.70	12679.15	92.011

a) Assuming the initial lead to have had the ratios $(^{206}Pb/^{204}Pb)_0 = 16.25$ and $(^{207}Pb/^{204}Pb)_o = 15.51$, compute the ^{206}Pb and ^{207}Pb ages for all samples. Use the equations listed in problem 3 above.

b) Calculate the $^{207}Pb^*/^{235}U$ and $^{206}Pb^*/^{238}U$ ratios for all samples.

c) Plot the ratios calculated in (b) on the concordia diagram constructed in 3 above.

d) Calculate the discordia using linear regression.

e) What is the estimated age of emplacement of the batholith?

f) What significance might a younger intrusion with a Rb-Sr date of 6.0×10^8 years have on the interpretation of the zircon data?

Notes on Linear Regression

Linear regression is a method used to find the equation of a line passing through data point in x - y space. The equation for a line in x - y space is:

$$y = mx + b$$

where m = slope of the line
and b = the y intercept.

If you have a number of x - y data points, x_i and y_i, then in order to perform linear regression you first make a table listing the values of the x_i and y_i values. You next determine the following sums:

Σx_i

Σy_i

$\Sigma(x_i y_i)$

$\Sigma(x_i^2)$

the slope, m, of the best fit line through the data points is then:

$$m = \{\Sigma(x_i y_i) - [(\Sigma x_i)(\Sigma y_i)]/N\}/[\Sigma(x_i^2) - (\Sigma x_i)^2/N]$$

and the y-intercept, b, is then:

$$b = \{[(\Sigma x_i][\Sigma(x_i y_i)]-[(\Sigma y_i)(\Sigma(x_i^2)]\}/[(\Sigma x_i)^2 - N[\Sigma(x_i^2)]\}$$

where N = the number of x - y pairs of data points.

Although this calculation can be done on a hand calculator, it is recommended that you do it using a computer spreadsheet program. Note that most spreadsheet programs have specific functions for doing linear regression that do the same thing as the above equations. You are free to use the built-in functions of your spreadsheet to do the linear regressions for this exercise.