# ON MICROPOLAR MODELLING OF CANCELLOUS BONE

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## INTRODUTION

Cancellous bone is a heterogeneous material with microstructural features. Microscopic and macroscopic continuum methods can be used for the mechanical modeling of such materials. The former method consists of the construction of a detailed (e.g. voxel based) Finite Element (FE) model that incorporates all the features of the local architecture of the trabeculae. This has the advantage that all microstructural effects are accounted for in a straightforward manner. However, large cancellous bone specimens lead to very time-consuming FE models. As an alternative, macroscopic continuum models can be used. Classical continuum theory is the simplest continuum model in which the stress at a material point depends only on the strain at the same point. As a consequence, it cannot incorporate microstructural size effects. To overcome this drawback, one may use higher-order continuum theories. Cosserat (i.e. micropolar) continuum theory is one of them. The present study addresses the application of micropolar continuum theory for the mechanical analysis of cancellous bone and the extraction of micropolar elastic moduli for cancellous bone by a micromechanical analysis.

## MICROPOLAR ELASTICITY

In addition to translations, micropolar continuum theory incorporates independent local rotations of material points. Consequently, material can transmit both Cauchy stresses as well as couple stresses. As a result, characteristic lengths appear in the constitutive equations. These characteristic lengths depend on the microstructure of material at hand.

The deformations for micropolar elasticity are given by

$$\varepsilon_{ij} = u_{j,i} - e_{ijk}\phi_k, \quad \kappa_{ij} = \phi_{j,i} \quad \text{with} \quad i, j, k = 1 \cdots 3 \qquad (1)$$

where $\varepsilon_{ij}$ denotes the micropolar strain tensor, $u_{j,i}$ the displacement gradients (where a comma indicates the partial derivative with respect to one of the spatial coordinates), $e_{ijk}$ the permutation tensor, $\kappa_{ij}$ the curvature tensor and $\phi_k$ the microrotation vector. The general form of the constitutive equations of a centro-symmetric (i.e. there is no coupling between Cauchy stresses and curvatures, nor between couple stresses and strains), linear elastic micropolar solid is

$$t_{ij} = C_{ijkl}\varepsilon_{kl}, \quad m_{ij} = D_{ijkl}\kappa_{kl} \qquad (2)$$

where $t_{ij}$ and $m_{ij}$ are the Cauchy and couple stress tensors, respectively, and the fourth order tensors $C_{ijkl}$ and $D_{ijkl}$ are called the micropolar stiffness tensors.

## CANCELLOUS BONE AS A MICROPOLAR CONTINUUM

Experimental evidence of micropolar elasticity in human compact bone has been reported by Lakes and co-workers [1,2]. The existence of micropolar effects for cortical bone in quasi-static bending has been demonstrated in [1]. The issue of application of higher-order continuum theories to mechanical analysis of bone (both cortical and cancellous) has been addressed by Fatemi *et al*. [3]. In [3], a simplified two-dimensional (2-D) bone-prosthesis configuration was analyzed using a micropolar-based FE formulation. Results of this analysis showed that the stress and strain intensities in the bone-prosthesis interface are different from those predicted by classical elasticity.

It is well known that the micromechanical effects are most important in regions of high strain gradients, e.g. near a hole or near a bone-implant interface [1]. To show these effects, we take the example of an artificial glenohumeral joint in which the cavity made in the glenoid is larger than the length of the prosthesis keel (see Figure 1a). To study the micromechanical effects on the stress concentration in cancellous bone near the hole, this bone-prosthesis system is modeled by a micropolar-based FE method. The prosthesis is assumed to be fully bonded to the bone. Displacements and microrotations are fixed to zero at the bottom and the prosthesis is loaded with a vertical force at the top (see Figure 1a). The prosthesis is assumed to be linear elastic, while the bone is taken to be strongly micropolar (see [3] for the specific material properties used for the bone and prosthesis). Figure 1b depicts the distribution of $t_{22}$ in the bone. Dark areas represent high stress regions. As expected, application of the micropolar - based FE model results in a 40% lower stress concentra-

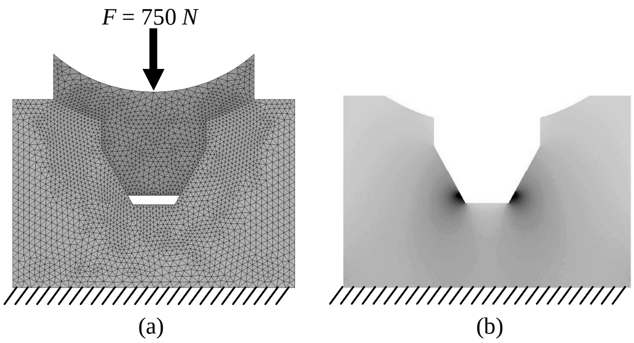


**Figure 1. (a) FE model of a bone-prosthesis system (b) Results: stress distribution in bone ($t_{22}$)**

tion at the bone-prosthesis interface near the hole, as compared to a classical elastic analysis [3]. Notice, for this example isotropic material properties have been assumed in combination with a strong micropolar effect. However, the micropolar elastic moduli ($C_{ijkl}$ and $D_{ijkl}$) must be evaluated from the microstructure of the material at hand. Since it is very hard to do this experimentally, in the following, a numerical identification framework is proposed to estimate the micropolar elastic moduli of a cancellous bone specimen.

## MICROPOLAR ELASTIC MODULI OF CANCELLOUS BONE

The issue of the identification of the micropolar elastic constants of cancellous bone in the context of micromechanical analysis has already been addressed by Fatemi *et al*. [4]. In this approach, it is assumed that at the microscopic level the bone tissue is an isotropic, Cauchy-type elastic material, whereas cancellous bone behaves as a homogeneous, anisotropic micropolar-type continuum at the macroscopic level. The effective elastic constants for the micropolar continuum were determined from the response of a bone specimen, whose microstructure was obtained from micro-CT scans (see Figure 2a) [4]. The identification procedure follows a rigorous homogenization approach and consists of the following steps: (i) Strains and curvatures are prescribed through the appropriate displacement and rotation boundary conditions. (ii) By equating the average work applied to the bone specimen with the macroscopic equivalent strain energy, the macroscopic stresses and couple stresses can be obtained from the reaction forces and moments at the boundary. (iii) Relating the applied strains to the stresses finally enables the evaluation of the effective elastic constants ($C_{ijkl}$ and $D_{ijkl}$).

This work [4] clearly showed that application of rotations at the boundary of the cancellous bone sample leads to boundary layer effects (Figure 2a: bending effects are high in dark regions). The thickness of this boundary layer is independent of the size of the material sample and is related to the microstructural length scale (e.g. the average size of the trabeculae). The boundary layer effects will result in a dependence of some of the effective properties $C_{ijkl}$ on the size of the material sample [5]. In addition, it is shown that the bending stiffnesses ($D_{ijkl}$) increase with the increase of the material sample size [5,6]. Clearly, this prohibits the evaluation of unique properties, since the material sample size is not a physical length scale in the problem. This led to the development of an alternative identification procedure, to be explained in the following.

Going back to the macroscopic problem (Figure 1a), it is clear from the specific loading configuration that the bone near the fully bonded bone-prosthesis interface is loaded in shear. This is known to lead to stiff boundary layers [7]. To account for these boundary layer effects on the effective properties of cancellous bone, an optimization-based identification procedure is proposed. The objective is to identify the effective elastic constants on the basis of the minimization of the

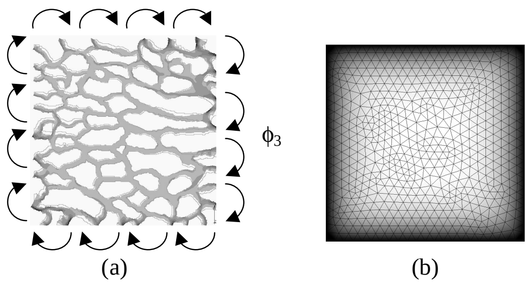


**Figure 2. Boundary layer - Application of microrotation at the boundary of (a) the micro-FE model of a cancellous bone specimen and (b) the equivalent micropolar FE model.**

difference between the total strain energy stored in the cancellous bone sample (see Figure 2a) and its equivalent micropolar continuum model (see Figure 2b). This minimization is performed on the sum of energy differences of a number of appropriate boundary value problems, leading to a fitted set of micropolar constants that can account for boundary layer effects. Due to the fact that the boundary effects (which are present in the microscopic continuum model, see e.g. Figure 2a) exist in the micropolar continuum model (Figure 2b) as well, it is anticipated that the estimation of the effective properties is size independent. Work is in progress to show this.

## DISCUSSION

The mechanical analysis of cancellous bone based on the micropolar continuum theory was addressed. The micropolar effects in regions near the bone-prosthesis interface lead to boundary layers and an associated reduction in stress peaks, which are not present in case of classical elastic behaviour. To account for these boundary layer effects , an optimization-based identification procedure was proposed as an alternative method for the more rigorous Cosserat homogenization approach [4,6].

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