SIMULATION OF BLOOD FLOW IN THE ABDOMINAL AORTA AT REST AND DURING EXERCISE USING A 1-D FINITE ELEMENT METHOD WITH IMPEDANCE BOUNDARY CONDITIONS DERIVED FROM A FRACTAL TREE

Brooke N Steele[1], Charles A Taylor[1, 2]

(1) Department of Mechanical Engineering
Stanford University
Stanford, CA

(2) Department of Surgery
Stanford University
Stanford, CA

INTRODUCTION

We have previously described a finite element method to solve the one-dimensional equations of blood flow [1]. This method was applied to predict flow distribution in vascular networks for simulation-based medical planning. This method was validated in a porcine thoracic aorto-aorto bypass model with a prescribed inlet flow and a simple resistance outlet boundary condition, but this model had only one outlet, and as a result the flow distribution was not sensitive to the choice of outflow boundary condition [2]. In general, outflow boundary conditions play a major role in blood flow distribution when modeling vascular networks with multiple branches. Olufsen described a novel approach to address this problem, namely a finite difference method to model blood flow in the major arteries coupled to a fractal tree outflow boundary conditions using Womersley's theory of vascular impedance [3].

Clearly, the utility of an outflow boundary condition is not manifest under a single physiologic state, e.g. resting conditions, but must mimic the physiologic changes due to varying demands of organs and tissues. For example, during lower limb exercise, the arterioles in the active muscles dilate and capillaries are recruited resulting in a decrease in the vascular resistance of the lower extremities. Conversely, arterioles in muscles and organs not involved in the performance of this task constrict resulting in an increase in the vascular resistance for these branches.

We describe the application of an impedance boundary condition derived from a fractal tree for modeling blood flow in the abdominal aorta during resting and exercise conditions. In this approach, the small arteries and arterioles are dilated or constricted, simulating the change in impedance during exercise. We then apply this method to simulate blood flow at rest and during exercise in 9 healthy subjects [4]. For each of these subjects, a baseline model is constructed that yields the flow distribution measured under resting conditions and results in physiologic pressures in the abdominal aorta. We then increased abdominal aorta inflow, constricted the vascular beds for the visceral organs and dilated the vascular beds for the lower extremities to simulate exercise in each subject.

METHODS

One dimensional analysis method

We solve the nonlinear one-dimensional equations of flow for a Newtonian fluid in an elastic vessel for area, $S(x,t)$, and flow, $Q(x,t)$. A constitutive equation relates pressure to area, $P(S(x,t),x,t)$. The flow rate is prescribed at the inlet and impedance boundary conditions are prescribed at each outlet. The resulting system of equations is solved using a space-time finite element method [1].

Impedance boundary condition

Impedance, Z, relates flow, Q, and pressure, P, in the frequency domain. In practice, we use the inverse of impedance, or admittance, $H(?)$ to compute flow rate from a time history of pressure. We convert admittance into an array in the time domain, $h[t]$, by taking the inverse Fourier transform.

$$h[t] = \mathcal{F}^{-1}\left(P(w) \Big/ Q(w)\right) \tag{1}$$

Convolution of the resulting inverse Fourier transform of the admittance with pressure yields flow at the time, $t = \Delta t \cdot n$ where n is the time step number.

$$q(n) = \sum_{j=0}^{N-1} h[j]p[n-j] \tag{2}$$

Here, N is the number of time steps in one cardiac cycle and the array, $p[n]$, represents the outflow pressure over the preceding cycle.

Baseline Model for Vascular Bed

Fractal trees, networks of self-similar bifurcating vessels, can be used to represent vascular beds [3]. In the present case, the trees are constructed for each outlet down to a minimum radius of $4\mu m$ using parameters obtained from the literature. The first parameter describes the area ratio between the parent and child vessels. At the junction, flow must be conserved. This leads to the branching relationship:

$$q_p = q_{d1} + q_{d2}; \quad r_p^k = r_{d1}^k + r_{d2}^k. \tag{3}$$

Observations show that k is neither constant nor organ specific. It has been suggested that there are three levels of arteries below the main branch vessel in which k varies from 2 to 3[4-7]. We prescribed k=2.5 for vessels with a radius larger than 250μm, k=2.76 for vessels with a radius greater than 50μm, and k=2.99 for vessels with a radius smaller than 50μm.

The next parameter used in building the fractal is the asymmetry ratio, ?, the relationship between the radii of the daughter vessels.

$$g = r_{d1}/r_{d2} \tag{4}$$

Studies have found that this parameter varies widely throughout vascular beds and is not organ specific [6]. To provide variation and minimize the level of generations, we assigned asymmetry ratios of 0.4, 0.6, and 0.9 respectively to the three levels described above.

The final parameter used to construct the fractal trees is the length-to-radius ratio of the vessels. Studies show that the length-to-radius ratios in a bed are highly variable, but the average values are organ specific [6-8]. We utilize the length-to-radius ratio to adjust the impedance to balance the organ specific flow distributions.

Modifications of Vascular Bed to Simulate Exercise

During exercise, the arterioles supplying the active muscles dilate and capillaries are recruited. The blood supply to these muscles can increase more than 10 fold. Similarly the blood supply to non-essential organs and inactive muscle is diminished to direct more of the cardiac output to high-demand locations. While at rest only 20-25% of muscle capillaries are in use, during exercise all of the capillaries can be recruited [9]. To mimic this behavior, we modify the radius of the vessels in the fractal tree to simulate the dilation or constriction of resistance vessels (r<300 μm) with an exercise factor. A factor less than one will constrict the resistance vessels and a factor greater than one will dilate the vessels.

Application to Blood Flow in Abdominal Aorta

The approach described above was used to assign outflow boundary conditions for a one-dimensional finite element model scaled to match the size of the abdominal aorta of each of 9 subjects imaged at rest and during lower limb exercise (Fig. 1). The model includes 12 vessels with one inlet boundary and 11 outlets. The inlet flow waveforms were taken from the PC-MRI velocity measurements obtained at the supra celiac level. The outlet boundary conditions were balanced by adjusting the length to radius ratios of the fractal trees to match velocity data obtained at the suprarenal and infrarenal levels and literature data, and to maintain a mean pressure of 85mmHg at the inlet. Once the appropriate boundary conditions were obtained for each subject under resting conditions, we applied the supraceliac flow measured during exercise at the inlet, constricted the 5 vessels with outlets above the aortic bifurcation and dilated the 6 vessels with outlets below the bifurcation. Flow measurements obtained during exercise both proximal and distal to the renal arteries and computed pressure were used to balance the amount of constriction and dilation of the distal beds.

Fig. 1. Model

RESULTS

We performed the simulations for all 9 subjects. Fig. 2 shows representative flow waveforms at the inlet, suprarenal, and infrarenal levels for measured and predicted values at rest and during exercise.

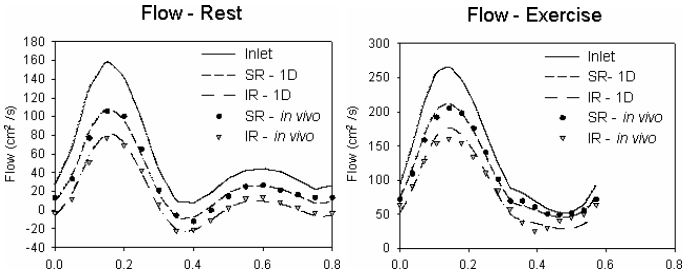

Fig. 2. Measured and predicted (1D) flow at the suprarenal (SR) and infrarenal (IR) levels at rest and during exercise. Resting boundary conditions were specified to give the best fit to the measured data. The distal vascular beds were constricted or dilated to simulate exercise.

DISCUSSION

We have described a method to prescribe outflow boundary conditions for a one-dimensional finite element method used to simulate blood flow under different physiologic states. Distal vessels in fractal tree models were constricted or dilated to simulate changes in the vascular beds during exercise for nine healthy subjects. Flow distributions between the subjects were varied and as a result, we have not yet developed a method to predict the exercise factors. Future work will include deriving a method to predict the exercise boundary conditions from resting data.

ACKNOWLEDGMENTS

The author's gratefully acknowledge the assistance of Mette Olufsen for providing methods to calculate impedance in fractal trees. Funding was provided by NSF ACI-0205741.

REFERENCES

1. Wan, J., B.N. Steele, S.A. Spicer, S. Strohband, G.R. Feijoo, T.J. Hughes, and C.A. Taylor, 2002, *A One-Dimensional Finite Element Method for Simulation-Based Medical Planning for Cardiovascular Disease.* Computer Methods in Biomechanics and Biomedical Engineering, Vol. 5, pp. 195-206.
2. Steele, B.N., J. Wan, J.P. Ku, T.J.R. Hughes, and C.A. Taylor, *In vivo Validation of a One-Dimensional Finite Element Method for Predicting Blood Flow in Cardiovascular Bypass Grafts. Accepted for publication in* IEEE Transactions in Biomedical Engineering.
3. Olufsen, M.S., 1999, *A Structured Tree Outflow Condition for Blood Flow in the Larger Systemic Arteries.* American Journal of Physiology, Vol. 276, pp. H257-H268.
4. Karch, R., F. Neumann, M. Neumann, and W. Schreiner, 2000, *Staged growth of optimized arterial model trees.* Annals of Biomedical Engineering, Vol. 28, pp. 495-511.
5. Rossitti, S. and J. Lofgren, 1993, *Optimality principles and flow orderliness at the branching points of cerebral arteries.* Stroke, Vol. 24, pp. 1029-1032.
6. Zamir, M., 1999, *On fractal properties of arterial trees.* Journal of Theoretical Biology, Vol. 197, pp. 517-526.
7. Iberall, A.S., 1967, *Anatomy and Steady Flow characteristics of the Arterial System with an Introduction to its Pulsatile Characteristics.* Mathematical Biosciences, Vol. 1, pp. 375-395.
8. Suwa, N., T. Nniwa, H. Fukasawa, and Y. Sasaki, 1963, *Estimation of Intravascular Blood Pressure Gradient by Mathematical Analysis of Arterial Casts.* Tohoku Journal of Experimental Medicine, Vol. 79, pp. 168-198.
9. Guyton, A.C.M.D., *Physiology of the Human Body*. 6 ed. 1984, San Francisco: Saunders College Publishing. 691.