MICROINDENTATION METHODS FOR EVALUATING NONLINEAR ELASTIC PROPERTIES OF ARTICULAR CARTILAGE

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INTRODUCTION

Microindentation has several advantages for the mechanical characterization of Articular Cartilage (AC) and other soft tissues. It requires minimal specimen preparation, can use mm-size specimens and can provide spatial distribution of properties with a resolution of ~10 microns. In contrast, conventional tests require specimens that have a standard shape, which means more extensive preparation, and complexities associated with grips or boundary interactions means that specimens are considerably larger. The net result is that conventional mechanical tests can measure properties averaged only on a mm-scale. The higher spatial resolution of Microindentation and its ability to probe small regions has two important consequences. First, the strong inhomogeneity of AC can be characterized. Second, specific regions near fissures or other lesions can be examined, and this means the degradation in mechanical properties associated with osteoarthritis can be quantified.

In an instrumented indentation test, the displacement of the indenter is measured continuously as a function of the applied force, both during loading and unloading [1]. The primary advantage over conventional hardness testing is that the elastic modulus can be evaluated from the force-displacement curve. Based on linear elastic solutions for blunt punches indenting a half-space, semi-empirical methods have been developed to obtain the linear elastic modulus from the stiffness of the unloading force-displacement curve [2]. However the method is not directly applicable to AC due to the following severe limitations: First, AC undergoes large deformations, hence finite elasticity is more appropriate. Second, Finite indentation depths are necessary to characterize AC, whereas existing methods are valid only for infinitesimal deformations. Finally, at sufficiently fast loading rates, the slope of the force-displacement curve immediately after unloading starts can be negative. In the existing method [2], this slope is proportional to the elastic modulus and hence the slope has to be positive. Thus, even if AC is treated as a linear elastic material, existing methods cannot be extrapolated to AC. A few previous works have examined the finite indentation contact problem using nonlinear hyperelastic models [3, 4]. Although Costa and Yin [4] identify some critical issues involved in extending the indentation method to soft tissues, they do not deduce explicit methods for parameter extraction. Hence, new microindentation methods are proposed for extracting the nonlinear elastic properties of AC and other soft tissues.

METHODS

In simple terms, the proposed method extracts material parameters by comparing the force-displacement predictions from model based simulations with measurements from an indentation test. The simulations are performed using the FEM package ABAQUS on a PC. A conical indenter with an apex angle of 67^o is treated as a rigid material. The cartilage is a 2mm thick layer lying on a rigid substrate. Frictionless boundary conditions are enforced between the indenter and cartilage as well as between the cartilage and substrate. The mesh is sufficiently refined to resolve the strongly varying stress fields under the conical tip. Cartilage is treated as an incompressible nonlinear hyperelastic material, and the following three strain energy densities are used:

Mooney-Rivlin: $$W_{MR}=C_1(I_1-3) + C_2(I_2-3) \quad (1)$$

Polynomial: $$W_{POLY} = D_1(I_1-3) + D_2(I_2-3)^2 \quad (2)$$

Exponential: $$W_{EXP} = B_1 [\exp(B_2(I_1-3)) -1] \quad (3)$$

Here W denotes the stored energy per unit volume, whereas I_1 and I_2 are invariants of the Cauchy-Green Stretch tensor. The material parameters are C_1 and C_2 for the MR material, D_1 and D_2 for POLY, and B_1 and B_2 for EXP. An asymptotic relation between the indentation force p and indenter displacement d is [3, 4]

$$p = (4/\pi) E \tan \alpha \, d^2 \quad (4)$$

where α is the indenter half-angle and the elastic modulus is given by

$$E_{MR}=4(C_1+C_2);\ E_{POLY}= 4D_1;\ E_{EXP} = 4B_1B_2 \quad (5)$$

Although these relations are valid only for small deformations, they are useful for characterizing measurements and guiding parameter extraction. **For brevity, the rest of this abstract will discuss only the MR model, but the talk will discuss all three material models.**

Steps involved in the parameter extraction are:

1. Establish a correlation $E=h(\beta)$. Fix $C_2/C_1=1/8$ and based on (4), fit quadratics $p=\beta d^2$ to curves predicted by FEM. Then relate known E to predicted β.
2. Minimize using Golden Section Search. For a given p-d measurement, obtain the fit coefficient β. The correlation $E=h(\beta)$ gives an initial estimate $\hat{E}$. Fix $(C_1+C_2)=\hat{E}/4$ and minimize error ratio ε (area between predicted and measured p-d curves divided by area under measured p-d curve). This gives optimal parameter ratios C_2/C_1.
3. Minimize using Simplex method. Using optimal ratios and $\hat{E}$ obtain three initial points. Minimize error ratio ε to obtain final values of (C_1, C_2).

Experiments were performed on bovine patellar cartilage using a Nanoindenter XP (MTS Inc.). Cartilage specimens had thickness ~2mm, surface area ~1cm^2 and were attached to a thin piece of subchondral bone. The bone was fixed to specimen holder with cyanoacrylate cement, and holder was rotated to orient the cartilage surface perpendicular to the indenter motion. Drops of PBS were frequently placed on the cartilage to prevent drying. Indentation tests were performed under load control using a 67^o conical diamond tip. The load increased at a constant rate (4mN/s) until the displacement reached a maximum, held constant for 10s and then decreased at the constant rate.

RESULTS

Although the predicted force-displacement curve agrees very well with a quadratic fit, it deviates from the asymptotic relation (4) (called lin in Figure 1a). The difference is small for indentation depths less than 0.1T (T denotes tissue thickness), but can be as large as 20% for indentation depths that approach 0.5T. This indicates the limitations of the asymptotic relation (4) for finite indentation of soft tissues. Using predicted p-d curves with fixed $C_2/C_1=1/8$, but over a range of values for E, the correlation h(β) was found to be E=1.048 β.

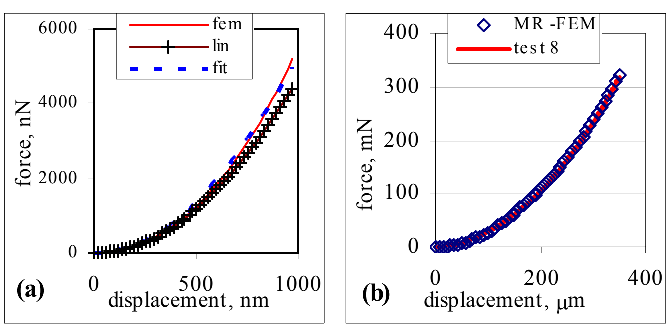

Figure 1 (a). Force-displacement curves with C_2/C_1=1/8 and E=4.8MPa for MR. (b) Comparison of predicted (MR-FEM) and measurements (test 8).

The parameter extraction method is validated as follows. The benchmark data was a p-d curve predicted by the FEM for C_1=1.067 and C_2=0.133 MPa. A quadratic curve-fit for $0\leq d<0.1T$ gave the fit-coefficient as β=4.58. Next, the correlation E=1.048β results in C_1+C_2=1.2 MPa. Starting with the range $2\leq C_1/C_2\leq 80$, the golden section search gave the optimum parameter ratio as C_1/C_2=7.99. Then the procedure takes the three initial points for the simplex to be (C_1,C_2) =(0.9603,0.1197), (0.9603,0.1463) and (1.1737,0.1463). Then, by minimizing the error to ε =1.8x10^{-5}, the MR parameters are extracted as C_1=1.070 and C_2=0.131 MPa. The error between the benchmark and extracted values are 0.28% for C_1, 1.50% for C_2 and 0.08% for E.

This procedure was then applied to measurements from five tests on bovine patellar cartilage. Table 1 shows the quadratic fit parameter β, the values of C_1+C_2 from the correlation E=h(β), the ratio C_2/C_1 obtained from golden section search, the final values extracted by simplex minimization – in columns labeled C_1 and C_2 - and the corresponding value of the error ratio. The average (st. dev.) values are C_1=0.496 (0.047), C_2=0.283 (0.013), E=3.115 (0.220) MPa and C_2/C_1=0.575 (0.045). The talk will discuss more extensive data as well as issues such as the resolution

Test no.	βx10^3	C_1+C_2	C_1/C_2	C_1	C_2	ε x 10^5
2	3.451	0.722	2.036	**0.555**	**0.278**	1.97
3	3.217	0.673	2.036	**0.538**	**0.303**	4.54
4	2.708	0.566	2.036	**0.453**	**0.275**	3.55
5	2.841	0.594	2.036	**0.458**	**0.271**	5.05
8	2.928	0.613	2.036	**0.474**	**0.289**	3.75

Table 1. MR Paremeter Extraction for bovine patellar AC

DISCUSSION

The primary goal was to develop methods to extract nonlinear hyperelastic parameters of AC from indentation data. The proposed methods, based on minimizing errors between predicted and measured data, extracted the benchmark values with errors of less than 1.5%. These values are well within the range of our experimental errors, suggesting that the proposed method can successfully extract nonlinear elastic parameters. The MR model has an extensive literature, and this was used to validate the accuracy of the FEM indentation results. Although it was not specifically developed to account for the structure of AC, it agrees remarkably well with measurements (error ε~10^{-5} in Table 1 and also see Figure 1b).

AC has a complex structure and its mechanical response clearly demonstrates dependence on loading-rate. The constitutive models considered here are purely elastic and cannot model such rate dependence. The rationale here is to adopt a simple continuum model, and thereby address the considerable complexities involved with the development and validation of the numerical model-based extraction method. In future, more elaborate models will be implemented to address loading-rate issues.

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