Homework #1: Answers

1. With reference to the home country’s trade triangle illustrated in figure 2.3, suppose the world relative price of clothing stays at the slope shown by the line $CED$. How would the home country’s volume of imports and exports be altered if:

![Figure 2.3](image)

a. a fire destroyed 10% of the clothing endowment? (3 points) or

![The figure illustrates that both imports and exports are smaller and, thus, the trade triangle is smaller.](image)
b. a bumper harvest expanded food production by 10%? (3 points)

The trade triangle is again smaller. This time because the country’s endowment of the importable good rose. Thus, in both cases, the country’s endowment approaches the country’s preferred consumption bundle at the given prices.

2. Referring to the previous exercise, if a fire destroys quantity $GE$ of clothing in Figure 2.3, will the home country cease to trade if the world relative price of clothing is shown (again) by the slope of the line $CED$? (3 points)

As the diagram illustrates, exploiting homotheticity, there is still positive trade. For trade to be extinguished, the endowment of $C$ would have to fall to point $H$. At that point, the endowment contains the same proportion of $F$ to $C$ that the country chooses to consume at given prices.
3. In a pair of diagrams such as Figures 2.3 and 2.4, illustrate the mutual gains from trade if:

a. tastes are similar between countries but endowments differ (3 points); and

![Diagram showing mutual gains from trade when tastes are similar but endowments differ.]

This diagram illustrates how two countries with the same tastes, but different endowments can trade to mutual gain. Note that, in this case, the countries will end up consuming more similar bundles of goods than they would in autarky. If preferences are homothetic, they will consume Food and Clothing in the same proportions. Can you explain why?

b. tastes are different but endowments are the same. (3 points)

![Diagram showing mutual gains from trade when tastes are different but endowments are the same.]

Note that in this case, the countries end up consuming more different bundles of goods than that (identical) bundle with which they were each endowed. Although the curves are somewhat dubiously drawn (especially the one with an intersecting pair) the diagram shows
balance trade, which is an essential property of equilibrium.

4. In Figure 2.5, a positively sloped curve is drawn to show the foreign supply of exports of food rising as the price of food rises. How can this response be reconciled with the chapter’s assumption that each nation’s production of commodities is fixed with respect to price? (3 points)

Recall that excess supply (i.e. export supply) is defined as \( E_s = \left( F - D_F \right) \), and that demand is itself a decreasing function of price (as well as an increasing function of income). Thus, as the relative price of food rises, and assuming that income effects are small in the neighborhood of the equilibrium, consumer demand falls. Combined with the fixed quantity of the food endowment, this implies an upward sloping export supply curve (at least until the income effect kicks in).

5. The individual whose tastes are shown by the indifference curve in Figure 2.6 is a net seller of food at autarky home prices, shown by line 1. This individual loses if trade with the rest of the world is allowed and food prices are lower there, shown by line 2. Show how this individual might gain if the world price of food is even lower than that shown by line 2.

The first arrow illustrates the drop in relative price from the internal trade price due to the opportunity to engage in international trade. Arrow two shows a second fall in price that is sufficiently large to restore the original level of welfare.
6. The relative price that clears the world’s food market is shown by line $0T$ in Figure 2.5.

a. Using the assumed balance in each country between total expenditures and total income, prove that the world's clothing market must be cleared as well. (3 points)

From total expenditures equals total income, and using a star to denote foreign values, we have:

\[ p^W_C D_C + p^W_F D_F = p^W_C \overline{C} + p^W_F \overline{F} \]  
\[ p^W_C D^*_C + p^W_F D^*_F = p^W_C \overline{C}^* + p^W_F \overline{F}^* \]  

But these can be rearranged to give trade balance conditions for each country:

\[ p^W_F (D_F - \overline{F}) = p^W_C (\overline{C} - D_C) \]  
\[ p^W_C (D^*_C - \overline{C}^*) = p^W_F (\overline{F}^* - D^*_F) \]  

(2)

The fact that the world's food market clears means that supply equals demand in the world food market, or that home excess demand equals foreign excess supply at world prices. That is:

\[ p^W_F (D_F - \overline{F}) = p^W_F (\overline{F}^* - D^*_F) \]  

(3)

But equations (2) and (3) together imply that:

\[ p^W_C (\overline{C} - D_C) = p^W_C (D^*_C - \overline{C}^*) \]  

(4)

Which is what we wanted to show.

b. Would this mutual clearing of markets take place if one country tried to live beyond its means? (3 points)

Now suppose that one country, say Home, seeks to import more food than the equilibrium quantity, diagram 2.5 shows that the price at which the Foreign country is willing to supply that greater quantity is above the price the Home country is willing to pay.
7. For the individual portrayed in figure 2.6, describe the trade pattern after the compensation scheme is in effect. How does this compare with the trade pattern of others in the country? (3 points)

Refering to figure 2.6, we can see that the individual in question is a net seller of food in autarky. With compensation, they act as if their endowment were at point $G$, and trade along a line parallel to 2 (i.e. the world price) but through point $G$. Under those conditions, the individual is a net buyer of food. Note that, if preferences are homothetic, since everyone will be consuming the same shares of food and clothing in autarky, the post-compensation equilibrium involves everyone holding the same bundle, thus this individuals trade position is just a scaled down version of that of the country as a whole and, since the country as a whole gains, so must the individual.

8. Suppose in each country there are fixed amounts of each of two types of laborers—those who grow food and those who make clothing.

a. Show how the trading pattern depends both on the relative supplies of the two factors in each country and upon their productivities in making outputs. (3 points)

Note that the country’s endowment of goods depends on the relative amount of each good produced. This, in turn, depends on the productivities of the workers as well as the relative supplies of the two factors.

b. If home labor of each type is less productive than its foreign counterpart, would the foreign country still gain from trade? Could the home country successfully compete with the foreign country? (3 points)

Yes. What matters is that the relative prices of the two goods in autarky differ across countries. If the prices are different, then food must be cheaper in one country and clothing in the other. Both countries can compete in one product, and both can gain from trade.
Workbook problems

1. Homothetic preferences: A family of indifference curves is defined as homothetic when the slopes of the indifference curves (i.e. the MRS) are identical along a ray from the origin. The indifference curves below are homothetic:

Assume homothetic preferences. Determine whether the countries will trade in each of the following circumstances: (6 total points, 3 for correct answers, 3 for correct explanations)

a. Countries have identical tastes and identical endowments.

There is no basis for trade in this case. Since the autarky endowments are the same, and tastes are the same, the autarky MRS is the same in each country. That is, the implicit autarky prices are identical, so there is no basis for trade.

b. Countries have identical preferences, their endowments differ and their endowments are not in the same ratio of food to clothing.

There is a basis for trade in this case, and it is illustrated above in 3.a.

c. Countries have identical preferences, and their endowments differ but the ratio of food to clothing is the same in the two countries.

There is no basis for trade in this case. Even though the endowments differ, because they are proportionally identical, and because preferences are homothetic, it will be the case that autarky relative prices are identical.

d. Countries have identical endowments but different preferences.

Trade is possible in this case. It is illustrated above in 3.b.

e. Countries have both different preferences and different endowments.

As long as the autarky prices are not identical there is a basis for trade. However, with both tastes and endowments differing, we cannot rule this possibility out a priori. Since this is unlikely, we would say that the answer is in general yes.

2. Autarky prices in the exchange model.

a. Consider a country blessed with an endowment of food and clothing. Explain how autarky relative prices are determined. In particular, how are prices affected by

i. the country’s preferences? (3 points)

For any given endowment, the stronger is a country’s preference for food relative to
clothing, the higher will be the autarky relative price. This follows from the fact, developed in class that, in equilibrium:

\[ MRS \equiv \frac{MU_F}{MU_C} = \frac{P_F}{P_C}. \]

ii. the country’s endowment. (3 points)

The larger the country’s endowment of food relative to clothing, the lower will be the price of food relative to clothing. This follows straightforwardly from the assumption of diminishing marginal rate of substitution and the fact that, in autarky, the endowed quantity is the quantity actually consumed.

b. Suppose this country opens up for international trade and that it imports food. Draw its budget line (you may choose some world relative price) and indicate the trade triangle in your diagram. (3 points)

Figure 2.3, reproduced on the first page of this document, does just this.
3. *Causes of higher welfare*. Is the following statement true or false? “If a small country is initially trading but stops doing so after being assigned a new endowment point, the real income of its single resident must have fallen. [Hint: you can prove/disprove this statement by the use of a single diagram.] (3 points)

Recall that the country is economically small, that is, it is a price taker. In illustrating the answer to this question, this means that the slope of the line reflecting price will be fixed. We now illustrate that this is false, because income may have risen or fallen. Consider the following picture.

Assuming homotheticity, and recalling that prices are fixed by the small country assumption, we know that the country will not trade from any endowment along a ray from the origin through its free trade consumption point. Thus, if the country is endowed with a point like 1, it will not trade, but will be better off than in the initial equilibrium (where it was endowed with E and traded to F). Similarly, if the country is endowed with a point like 2, it will not trade, but it will be worse off than in the initial equilibrium.
4. Using the Edgeworth box diagram.

**a. (3 points)**

i. The region of mutual advantage is the shaded lens shaped area defined by the two indifference curves passing through the endowment point, E.

ii. If these are homothetic preferences, the contract curve is the straight line 00*. Recall that the contract curve is the locus of tangencies between indifference curves, and if they are symmetric about the main diagonal, as drawn, their locus of tangencies will be the main diagonal.

iii. The Home country will export clothing and import food.

**b. (3 points)**

i. All gains from trade are exhausted at E—i.e. the initial endowment point is in the pareto set.

ii. The preferences are the same as in a, and so the contract curve is still the diagonal.

iii. There is no trade in this case.
c. (3 points)

i. The shaded area.

ii. The straight line connecting the home and foreign origins.

iii. Home exports food and imports clothing.

d. (3 points)

i. The shaded area.

ii. The straight line connecting the home and foreign origins.

iii. Home exports food and imports clothing.
e. (3 points)

i. The region of mutual advantage is the shaded area in the above diagram.

ii. The contract curve coincides with the home country clothing axis and foreign country food axis.

iii. The home country imports clothing and exports food.

f. (3 points)

i. The region of mutual advantage is the rectangular lightly lined area, enclosed by the home and foreign indifference curves.

ii. The more heavily shaded area is the contract “curve”.

iii. The home country imports food and exports clothing.